Mixed Cayley graphs of diameter two of order asymptotically approaching the Moore bound

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For every $n \geq 1$ there exists a group H_n of order $|H_n| = 2^{2n}(2^{2n} - 1)$ and a symmetric generating set U_n of size $\Delta_n = 2^{2n} + 2^{n+2} - 6$ in H_n such that the (undirected) Cayley graph $C(H_n, U_n)$ has diameter 2.

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Note: This replacement works for all n, but we focus on the Cayley case.

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For every c such that $0 \leq c \leq +\infty$ there exists an infinite sequence of mixed (Δ_n, d_n)-regular Cayley graphs G_n of diameter 2 such that $|G_n|/M_2(\Delta_n, d_n) \to 1$ and $\Delta_n/d_n \to c$ as $n \to \infty$.

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A full extension of Theorem 2 to simple and irredundant mixed Cayley graphs remains open – lack of suitable generating sets for Cayley graphs and digraphs approaching the Moore bound for diameter 2.

The end

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