Decomposition of Complete Bipartite Graphs into Prisms

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Decompositions and factorizations

Definition

A G-decomposition of the complete graph K_n or $K_{n,n}$ is a collection of subgraphs $G_1, G_2, \ldots, G_s$, all isomorphic to G, such that every edge of K_n or $K_{n,n}$ belongs to exactly one copy G_i of G.

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Definition

A k-factorization of K_n or $K_{n,n}$ is a G-factorization where G is a k-regular subgraph of K_n or $K_{n,n}$.

k-factorizations and decompositions of K_n

$k = 1$ 1-factorizations (only)

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- 2-factorizations: Oberwolfach Problem OP$(s_1, s_2, \ldots, s_t)$ (n odd)
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$OP(s_1, s_2, \ldots, s_t)$: Let $n = s_1 + s_2 + \cdots + s_t$.

We decompose K_n into $(n-1)/2$ copies of $C_{s_1} \cup C_{s_2} \cup \cdots \cup C_{s_t}$.

Interpretation: We have a conference with n mathematicians, and we want to sit them over $(n-1)/2$ nights around a collection of round tables of sizes $s_1, s_2, \ldots, s_t$ so that each of them sits next to each other exactly once during the conference.

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Spouse avoiding version: Use $K_n - M$ when n is even. M is a perfect matching.

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Alspach, Gavlas (2001); Sajna (2002)

A C_m-decomposition of K_n exists for all admissible values of n.

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K_{k_1,k_2} can be decomposed into C_n if and only if n, k_1, k_2 are all even, n divides $k_1 k_2$ and both $k_1, k_2 \geqslant \frac{n}{2}$.

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$k = 3$ cubic graph factorizations

The necessary condition is that $n = 6s + 4$.

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Conjecture

All cubic graphs of order $n = 6s + 4$ for $s \geq 3$ factorize K_n.

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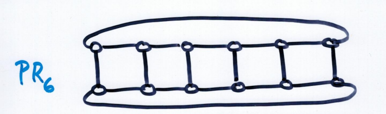

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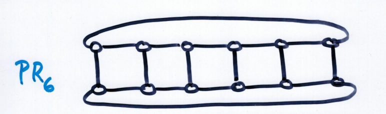

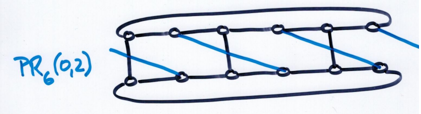

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Maheo (1980), Kotzig (1981)

$Q_3 = PR_4$ decomposes K_n if and only if $n \geq 16$ and either
(i) $n \equiv 1 \pmod{24}$ or
(ii) $n \equiv 0 \pmod 8$ and $n \equiv 1 \pmod 3$.

Decomposition tools

Definition

An α-labeling of a bipartite graph G with m edges is an injection f from $V(G) = X \cup Y$ to $\{0, 1, \ldots, m\}$ such that the set of all edge lengths (defined as $\ell(xy) = f(y) - f(x)$) is equal to the set $\{1, 2, \ldots, m\}$ and there is λ such that $f(x) \leq \lambda$ and $f(y) > \lambda$ for every edge $e = xy$.

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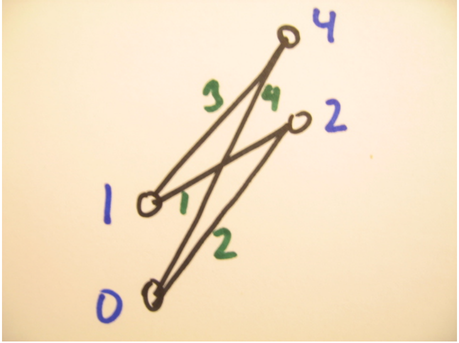

k-factorizations and decompositions of K_n

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Frucht, Gallian (1988)

All prisms PR_m with $3m$ edges are graceful and therefore decompose K_{6m+1}. When m is even, then PR_m decompose K_{6mk+1} for any $k > 0$.

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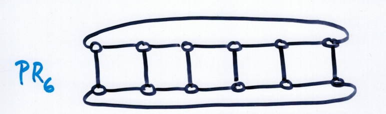

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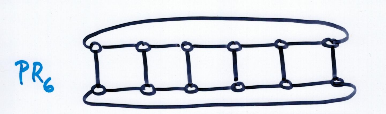

Cichacz, Dib, Froncek (2013)

$PR_m(0, 2)$ with $3m$ edges decomposes K_{6mk+1} for any $k > 0$.

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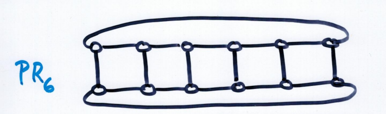

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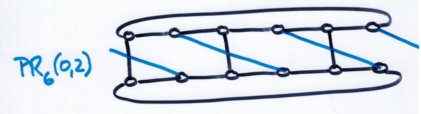

$PR_6(0,j)$- and $PR_8(0,j)$-decompositions of K_n

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Cichacz, Froncek, Meszka (2011)

PR_6- and $PR_6(0,2)$-decomposition of K_n exists iff $n \equiv 1, 28 \pmod{36}$.

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PR_8-, $PR_8(0,2)$-, and $PR_8(0,4)$-decomposition of K_n exists iff $n \equiv 1, 16 \pmod{48}$.

$PR_n(0, j)$-factorization of $K_{n,n}$

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Main ingredients:

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Main ingredients:

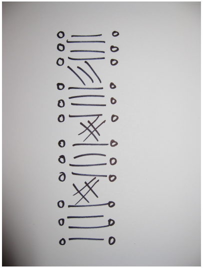
Bermond, Huang (1978); Sotteau (1981)

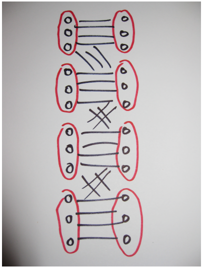
K_{k_1,k_2} can be decomposed into C_n iff n, k_1, k_2 are all even, $k_1 \geq k_2 \geq n/2$, and $n | k_1 k_2$.

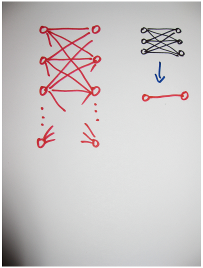
Corollary

$K_{n,n}$ can be factorized into C_{2n} for n even.

$PR_n(0, j)$-factorization of $K_{n,n}$







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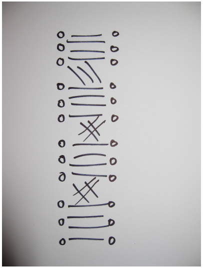
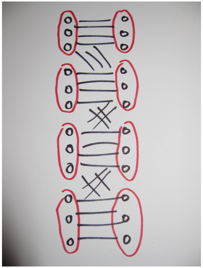
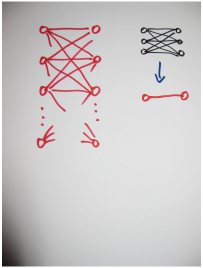

Now factorize $K_{n/3,n/3}$ into Hamiltonian cycles $C_{2n/3}$ (remember $n/3$ is even).

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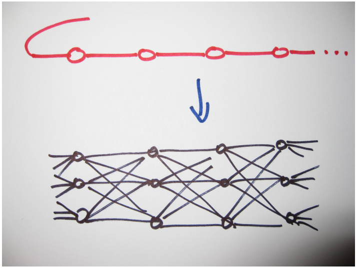

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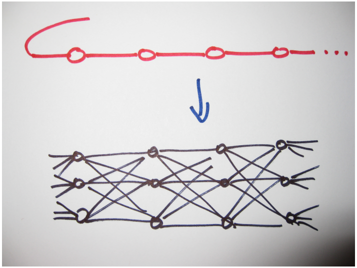

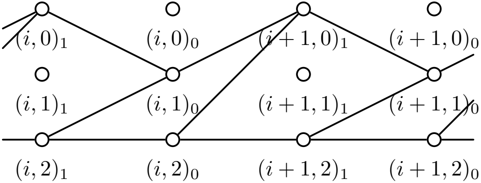

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Open problems

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Cichacz, Froncek, (June 2014)

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$PR_n(0,j)$-decomposition of $K_{3n/2,3n/2}$

Necessary conditions: $n \equiv 0 \pmod 4, j \equiv 0 \pmod 2$

Open problems

Decompositions of $K_{3n/2,3n/2}$ when

- $n \equiv 0 \pmod 8, j \equiv 2 \pmod 4$ and and $n/\gcd(n,j) \equiv 1 \pmod 2$
- $n \equiv 4 \pmod 8, j \equiv 2 \pmod 4$

Cichacz, Froncek, (June 2014)

$PR_n(0,j)$-decomposition of $K_{3n/2,3n/2}$ exists when $n \equiv 0 \pmod 8$.

Recall:

Cichacz, Froncek, Kovar (2009)

$PR_n(0,j)$-decomposition of $K_{3n/2,3n/2}$ exists when $n \equiv 0 \pmod 8$ and $n/\gcd(n,j) \equiv 0 \pmod 2$.

$PR_n(0,j)$-decomposition of $K_{3n/2,3n/2}$

Cichacz, Froncek, (June 2014)

$PR_n(0,j)$-dec. exists when $n \equiv 0 \pmod 8$.

Cichacz, Froncek, Kovar (2009)

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$PR_n(0, j)$-dec. exists when $n \equiv 0 \pmod 8$ and $n/\gcd(n, j) \equiv 0 \pmod 2$.

Observation

$PR_n(0, j)$ is isomorphic to $PR_n(0, -j)$.

$PR_n(0,j)$-decomposition of $K_{3n/2,3n/2}$

Cichacz, Froncek, (June 2014)

$PR_n(0,j)$-dec. exists when $n \equiv 0 \pmod 8$.

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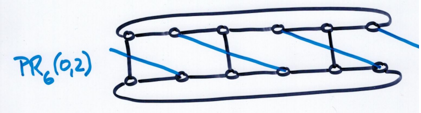
$PR_n(0,j)$ is isomorphic to $PR_n(0,-j)$.



$PR_n(0,j)$-decomposition of $K_{3n/2,3n/2}$

Cichacz, Froncek, (June 2014)

$PR_n(0,j)$-dec. exists when $n \equiv 0 \pmod 8$.

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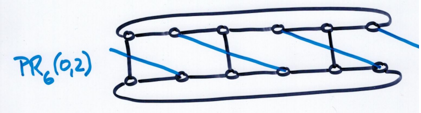
Observation

$PR_n(0,j)$ is isomorphic to $PR_n(0,-j)$.



Observation

When $n/\gcd(n,j) \equiv 1 \pmod 2$, then $n/\gcd(n,-j) \equiv 0 \pmod 2$.

$PR_n(0,0)$-decomposition of $K_{m,m}$

$PR_n(0,0)$-decomposition of $K_{m,m}$

DF (2015)

$PR_n(0,0)$-decomposition of $K_{m,m}$ exists iff $m \equiv 0 \pmod 6$ and $3n | m^2$.

$PR_n(0,0)$-decomposition of $K_{m,m}$

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How we prove it???

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How we prove it???

Similar to the *speed dating problem*

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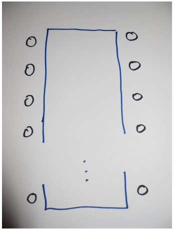

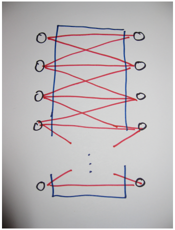

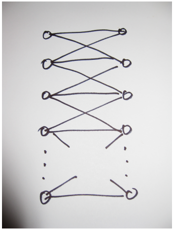

$PR_n(0,0)$-decomposition of $K_{m,m}$

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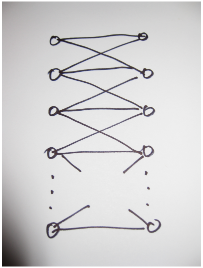

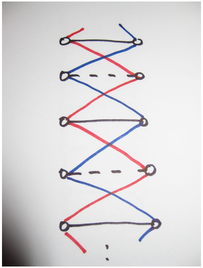

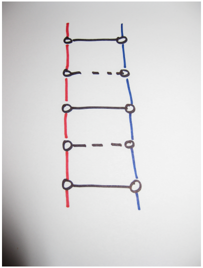

$PR_n(0,0)$-decomposition of $K_{m,m}$

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$PR_n(0,0)$-decomposition of $K_{m,m}$

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$PR_n(0,0)$-decomposition of $K_{m,m}$ exists iff $m \equiv 0 \pmod 6$ and $3n | m^2$.

Step 1: Squeeze $K_{m,m}$ into $K_{m/3,m/3}$.

$PR_n(0,0)$-decomposition of $K_{m,m}$

DF (2015)

$PR_n(0,0)$-decomposition of $K_{m,m}$ exists iff $m \equiv 0 \pmod 6$ and $3n | m^2$.

Step 1: Squeeze $K_{m,m}$ into $K_{m/3,m/3}$.

Step 2: Decompose $K_{m/3,m/3}$ into an appropriate graph H.

$PR_n(0,0)$-decomposition of $K_{m,m}$

DF (2015)

$PR_n(0,0)$-decomposition of $K_{m,m}$ exists iff $m \equiv 0 \pmod 6$ and $3n | m^2$.

Step 1: Squeeze $K_{m,m}$ into $K_{m/3,m/3}$.

Step 2: Decompose $K_{m/3,m/3}$ into an appropriate graph H.

Step 3: Blow up H into $H[3K_1]$ (and $K_{m/3,m/3}$ back into $K_{m,m}$).

$PR_n(0,0)$-decomposition of $K_{m,m}$

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Step 4: Decompose $H[3K_1]$ into $PR_n(0,0)$.

$PR_n(0,0)$-decomposition of $K_{m,m}$

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For $m \geq 3n/2$ we have $H \cong C_n$.

$PR_n(0,0)$-decomposition of $K_{m,m}$

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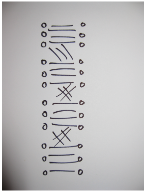
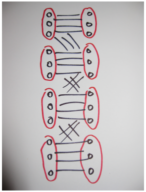
Step 4: Decompose $H[3K_1]$ into $PR_n(0,0)$.

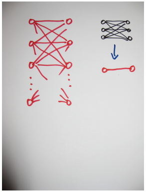
For $m \geq 3n/2$ we have $H \cong C_n$.

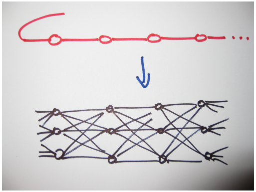
For $m < 3n/2$ we have $H \cong C_{2m/3} \cup M$, where $C_{2m/3}$ is a Hamiltonian cycle in $K_{m/3,m/3}$ and M is a matching of size t such that $t|m$.

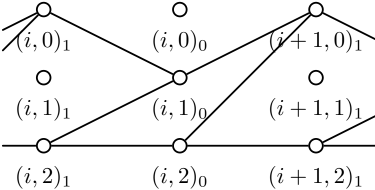
$PR_n(0,0)$-decomposition of $K_{m,m}$









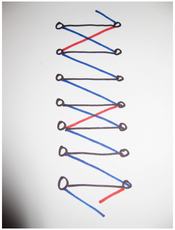


$PR_n(0,0)$-decomposition of $K_{m,m}$

Step 2: Decompose $K_{m/3,m/3}$ into an appropriate graph H.

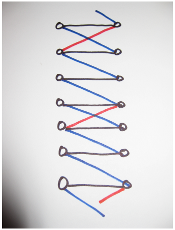
$PR_n(0,0)$-decomposition of $K_{m,m}$

Step 2: Decompose $K_{m/3,m/3}$ into an appropriate graph H.



$PR_n(0,0)$-decomposition of $K_{m,m}$

Step 2: Decompose $K_{m/3,m/3}$ into an appropriate graph H.

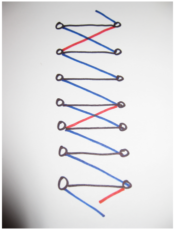


DF (June 1, 2014, near Orvieto)

H-decomposition of $K_{m/3,m/3}$ exists whenever $m \equiv 0 \pmod 6$ and $3n | m^2$.

$PR_n(0,0)$-decomposition of $K_{m,m}$

Step 2: Decompose $K_{m/3,m/3}$ into an appropriate graph H.



DF (June 1, 2014, near Orvieto)

H-decomposition of $K_{m/3,m/3}$ exists whenever $m \equiv 0 \pmod 6$ and $3n | m^2$.

DF (2015)

$PR_n(0,0)$-decomposition of $K_{m,m}$ exists iff $m \equiv 0 \pmod 6$ and $3n | m^2$.

$PR_n(0,j)$-decomposition of $K_{m,m}$

$PR_n(0, j)$-decomposition of $K_{m,m}$

DF (2015)

$PR_n(0, 0)$-decomposition of $K_{m,m}$ exists iff $m \equiv 0 \pmod 6$ and $3n | m^2$.

$PR_n(0, j)$-decomposition of $K_{m,m}$

DF (2015)

$PR_n(0, 0)$-decomposition of $K_{m,m}$ exists iff $m \equiv 0 \pmod 6$ and $3n|m^2$.

Corollary (DF and S. Cichacz, June 2014)

$PR_n(0, 2)$-decomposition of $K_{m,m}$ exists when $m \equiv 0 \pmod{12}$ and $3n|m^2$.

$PR_n(0, j)$-decomposition of $K_{m,m}$

DF (2015)

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$PR_n(0, 2)$-decomposition of $K_{m,m}$ exists when $m \equiv 0 \pmod{12}$ and $3n | m^2$.

Step 1: Squeeze $K_{m,m}$ into $K_{m/2,m/2}$. Note that $m/2 \equiv 0 \pmod 6$.

$PR_n(0,j)$-decomposition of $K_{m,m}$

DF (2015)

$PR_n(0,0)$-decomposition of $K_{m,m}$ exists iff $m \equiv 0 \pmod 6$ and $3n|m^2$.

Corollary (DF and S. Cichacz, June 2014)

$PR_n(0,2)$-decomposition of $K_{m,m}$ exists when $m \equiv 0 \pmod{12}$ and $3n|m^2$.

Step 1: Squeeze $K_{m,m}$ into $K_{m/2,m/2}$. Note that $m/2 \equiv 0 \pmod 6$.

Step 2: Decompose $K_{m/2,m/2}$ into $PR_n(0,0)$.

$PR_n(0,j)$-decomposition of $K_{m,m}$

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Step 2: Decompose $K_{m/2,m/2}$ into $PR_n(0,0)$.

Step 3: Blow up $PR_n(0,0)$ into $PR_n(0,0)[2K_1]$ (and $K_{m/2,m/2}$ into $K_{m,m}$).

$PR_n(0, j)$-decomposition of $K_{m,m}$

DF (2015)

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Corollary (DF and S. Cichacz, June 2014)

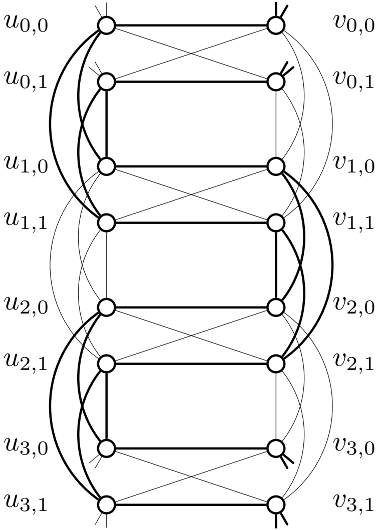
$PR_n(0, 2)$-decomposition of $K_{m,m}$ exists when $m \equiv 0 \pmod{12}$ and $3n|m^2$.

Step 1: Squeeze $K_{m,m}$ into $K_{m/2,m/2}$. Note that $m/2 \equiv 0 \pmod 6$.

Step 2: Decompose $K_{m/2,m/2}$ into $PR_n(0, 0)$.

Step 3: Blow up $PR_n(0, 0)$ into $PR_n(0, 0)[2K_1]$ (and $K_{m/2,m/2}$ into $K_{m,m}$).

Step 4: Decompose $PR_n(0, 0)[2K_1]$ into $PR_n(0, 2)$.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$v_{3,0}$
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$PR_n(0, j)$-decomposition of $K_{m,m}$

DF (2015)

$PR_n(0, 0)$-decomposition of $K_{m,m}$ exists iff $m \equiv 0 \pmod 6$ and $3n|m^2$.

Corollary (DF and S. Cichacz, June 2014)

$PR_n(0, 2)$-decomposition of $K_{m,m}$ exists when $m \equiv 0 \pmod{12}$ and $3n|m^2$.

$PR_n(0,j)$-decomposition of $K_{m,m}$

DF (2015)

$PR_n(0,0)$-decomposition of $K_{m,m}$ exists iff $m \equiv 0 \pmod 6$ and $3n|m^2$.

Corollary (DF and S. Cichacz, June 2014)

$PR_n(0,2)$-decomposition of $K_{m,m}$ exists when $m \equiv 0 \pmod{12}$ and $3n|m^2$.

Corollary (DF and S. Cichacz, June 2014)

$PR_n(0,6)$-decomposition of $K_{m,m}$ exists when $m \equiv 0 \pmod{12}$ and $3n|m^2$.

Thank you!