Techniques for Constructing Small Regular Graphs of Given Girth and Related Topics

Domenico Labbate

Dipartimento di Matematica, Informatica ed Economia
Università degli Studi della Basilicata – Potenza (Italy)

email: domenico.labbate@unibas.it

The *Cage Problem* asks for the construction of regular simple graphs with given degree and girth and minimum order. A (k, g)*–graph* is a k–regular graph of girth g. Sachs proved in 1963 that (k, g)–graphs exists for each $k \geq 3$ and $g \geq 5$. *Moore's bound* is obtained when counting the minimum number of vertices necessary to construct a (k, g)–graph. A (k, g)–graph whose order attains Moore's bound is, by definition, also a *Moore graph*. It is well known that the Moore graphs exist for girth 5 and $k = 2, 3, 7$ and maybe 57 and girth $6, 8$ or 12 and they are incidence graphs of finite projective planes, generalized quadrangles or generalized hexagons, respectively. Moreover Hoffman, Singleton, Feit, Higman, Damerell, Bannai and Ito proved in the 60–70's that there are no further Moore graphs.

Thus, it is natural to approach the more general problem of determining the minimum order of (k, g)–graphs. We denote this minimum value by $n(k, g)$ and a graph attaining this minimum value is said to be a (k, g)*–cage*. Hence, in most cases the number of vertices in a (k, g)–cage is strictly greater than Moore's bound. Several authors are trying to construct (k, g)–cages, or at least smaller (k, g)–graphs than previously known ones.

In this talk, we will describe several techniques (algebraical, geometrical and purely combinatorial) that we used to construct small regular graphs of girth 5, 6, 7 and 8 as well as to solve some related problems such as the existence of symmetric configurations and the search for C_4–free graphs of large size. In particular, we will point out how these techniques are related and how they are helpful in solving the above mentioned problems. Moreover, we will present new and recent results obtained for small regular (k, g)–graph of girth 7 and 8 and for biregular $(\{r, m\}, g)$–graphs of girth 5. Finally, we will present some possible further developments of this topic.