Enumeration Of $(3, g)$ Hamiltonian bipartite graphs upto $2^{3g/4}$ vertices

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Abstract

We present a range of $(3, g)$ Hamiltonian bipartite graphs for a range for vertices for even values of girth g satisfying $6 \leq g \leq 16$ and present a new methodology to analyze the trivalent cage problem for even girth. Our lists of $(3, g)$ Hamiltonian bipartite graphs have been found to be significantly more dense than other known lists of $(3, g)$ graphs, and hence allow confirmation of the $(3, g)$ upper bounds. This would be even more useful for the Cage Problem if these general computational approaches could be made to work for higher values of girth.

We pose a problem of enumerating $(3, g)$ Hamiltonian bipartite graphs upto $2^{3g/4}$ vertices that is motivated by the open problem, "Finding an infinite family of trivalent graphs with large girth g and order 2^{cg} for $c < 3/4$."

We also introduce $(3, g)$ sub-problems as follows. We decompose the problem of finding the smallest $(3, g)$ Hamiltonian bipartite graph to sub-problems of for finding $(3, g)$ Hamiltonian bipartite graph with symmetry factor b having the minimum number of vertices. Symmetry factor is a parameter that reflects the extent of rotational symmetry.

The enumeration of $(3, g)$ Hamiltonian bipartite graphs with symmetry factor b upto $2^{3g/4}$ vertices corresponds to the sub-problem for finding $(3, g)$ Hamiltonian bipartite graph with symmetry factor b having the minimum number of vertices.

We consider the enumeration of $(3, g)$ Hamiltonian bipartite graphs upto $2^{3g/4}$ to be exhaustive if all even vertices less than or equal to $2^{3g/4}$ that have a $(3, g)$ Hamiltonian bipartite graph are listed, with proof for non-existence for vertices not listed, and at least one $(3, g)$ Hamiltonian bipartite graph for each of the vertices listed. If list of vertices for which $(3, g)$ Hamiltonian bipartite graphs upto $2^{3g/4}$ is not complete, then it is a partial list. We consider two cases of partial enumeration, where results on existence $(3, g)$ Hamiltonian bipartite graph for some vertices listed in specified range are inconclusive, but upper bound for $(3, g)$ for even girth g can be confirmed from the list of $(3, g)$ Hamiltonian bipartite graphs.

We show that $(3, 6)$ Hamiltonian bipartite graphs exist for all even vertices greater than equal to 14. We confirm that there exists only one Hamiltonian bipartite $(3, 6)$ cage by enumeration and isomorphism checking for 14 vertices, girth 6 and symmetry factor 7.

We show that $(3, 8)$ Hamiltonian bipartite graphs exist for all even vertices between 30 and 90, with the exception of 32, for which we show that a $(3, 8)$ Hamiltonian bipartite graph with 32 vertices does not exist. We resolve sub-problems for $(3, 8)$ Hamiltonian bipartite graphs and find the minimum number of vertices for $(3, 10)$

Hamiltonian bipartite graphs for all symmetry factors and find the known $(3, 8)$ cage, Tutte-Coxeter graph. We also confirm that there exists only one Hamiltonian bipartite $(3, 8)$ cage.
We resolve sub-problems for $(3, 10)$ Hamiltonian bipartite graphs and find the minimum number of vertices for $(3, 10)$ Hamiltonian bipartite graphs for all symmetry factors except 35, and find a known $(3, 10)$ cage, Harris graph with symmetry factor 7. We also confirm that there exists exactly one Hamiltonian bipartite $(3, 10)$ cage with symmetry factor 7.
We resolve sub-problems for $(3, 12)$ Hamiltonian bipartite graphs for symmetry factors $3, 4, 5, 6, 7, 9$ and find the minimum number of vertices for $(3, 12)$ Hamiltonian bipartite graphs these symmetry factors find the known $(3, 12)$ cage, Tutte-12 cage for symmetry factor 9, and the best upper bound for other symmetry factors. We also confirm that there exists exactly one Hamiltonian bipartite $(3, 12)$ cage with symmetry factor 9.
We resolve sub-problems for $(3, 14)$ Hamiltonian bipartite graphs for symmetry factors $5, 6$ and find the minimum number of vertices for $(3, 12)$ Hamiltonian bipartite graphs these symmetry factors find the known $(3, 14)$ record graph found by Exoo for symmetry factor 8, and the best upper bound for other symmetry factors.