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Title: Graphs similar to strongly regular graphs

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The degree/diameter problem is the problem of finding the largest possible graph with given diameter $d$ and given maximum degree $k$. For graphs with diameter 2 the upper bound (Moore bound) is simplified to $k^2 + 1$. In 1980 Erdös, Fajtlowicz and Hoffman showed that, with the exception of the cycle of length 4, there does not exist any $k$-regular graph with diameter 2 and $k^2$ vertices (such graph has order one less than the Moore bound). Authors reduced this problem to solving the matrix equation

$$A^2 + A - (k-1)I = J + K,$$

where $A$ is the adjacency matrix of the graph, $I$ is the identity matrix, $J$ is the all-ones matrix and $K$ is the matrix of a suitable 1-factor.

Our aim is to solve the generalisation of the previous problem to one in which we replace Moore graphs with diameter 2 by strongly regular graphs. That is, we are looking for $k$-regular graphs on $n$ vertices such that their adjacency matrix $A$ satisfies the equation

$$A^2 + (c-a)A + (c-k)I = cJ + K.$$

We derive necessary conditions for parameters $(n, k, a, c)$ analogous to the integral criterion for strongly regular graphs. In this process the systemic application of algebraic properties of the third power of adjacency matrix $A^3$ proves to be crucial. Finally we find the complete (infinite) list of parameters satisfying these necessary conditions. Existence of graphs with these parameters remains an open problem.