On graphs of large size without small cycles and commutative diagrams and their applications

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We define a cycle indicator of a vertex of a simple graph as a minimal cycle through the vertex. A cycle indicator of a graph is a maximum of cycle indicator of their vertices. The maximal size of the graph with a given cycle indicator is evaluated. This bound turns out to be sharp in difference with the Even Circuit Theorem by P. Erdős and its corollary for graphs of given girth. The sharpness is proven explicitly by a construction of the family of small world graphs with increasing cycle indicator, such that their magnitude is on a new bound.

Let us refer to a directed graph Γ as balanced directed graph if it is a graph without multiple arrows such that numbers of inputs and outputs are the same for every vertex.

The class of a balanced directed graphs is an extension of the class of simple graphs for which the concept of a girth can be naturally defined. We evaluate precisely the maximal size of balanced directed graph on v vertices of girth $> d$.

Concepts of a family of small world graphs and a family of graphs of large girth can be generalized on a class of balanced directed graphs.

We prove, that for each pair (K, S), where K is commutative ring and S be its multiplicatively closed subset without zero, there exists an infinite directed regular balanced graph $\Gamma_S(K)$ without commutative diagrams.

We will use well defined functor $(K, S) \to \Gamma_S(K)$ for the construction of families of graphs of large girth, graphs with large cycle indicator, small world graphs for which $\Gamma_S(K)$ will appear as well defined projective limit.

The brief survey of applications of $\Gamma_S(K)$ to Information Security will be given.