

# Applying Sperner antichain to digital fingerprint detection

Ben-shung Chow

National Sun Yet-sen University [bschow@mail.ee.nsysu.edu.tw](mailto:bschow@mail.ee.nsysu.edu.tw)

## Abstract

Sperner family, formally an antichain in the inclusion lattice over the power set of a universal set  $X$ , is also called an independent system. The independence is defined as the non-containing-ship between every pair of members. In other words, the dependence is defined as the existence of a containing-ship for some pairs. This is a relation between two members. In contrast, the dependence in linear algebra is defined by the relation between one member and one group (many members). We therefore ask if this relational difference for the Sperner family is appropriated. Is this dependence relation defined for the Sperner member too strict?

An independent system (the Sperner family) is interpreted by us to have no redundant member in the family. Redundant member is clearly understood by words is a member, who does make any difference for the family if he exists or not. By this interpretation, the dependence relation is built upon the redundant member with the rest of the family. However, we shall prove this relation finally becomes to the personal relation between two. To check if there is a difference made by the suspicious redundant member, the originally “static” member needs to be regarded as an operator to have the ability to influence. One simple arrangement is to interpret the family (union of members) as a Boolean operator composed a sum (logic OR) of products (logic AND). For example, the family  $\{[11000], [10100], [01100]\}$  is regarded as the operator  $ab + ac + bc$ .

Under the above operator interpretation (interpretation 2, relative to the interpretation 1 about redundancy), two proofs are developed for the goal that the interpretation 1 leads to the original definition of containing-ship between two members used in the Sperner family. One is proved by truth table method. We also visualize this method by designing a full-pattern (the all possible inputs in the truth table) image to be processed by the family operator and the family minus one operator ( $ab + ac + bc$  vs.  $ab + ac$  for example) to check the difference. The second proof is by transforming the logic operation to propositional calculus.

Using this interpretation, the Sperner family can be easily extended to many applications for a compact purpose. In order to control the redistribution of content, digital fingerprinting is used to trace the consumers who use their content for unintended purposes [1-4]. These fingerprints can be embedded in multimedia content through a variety of watermarking techniques. Conventional watermarking techniques are concerned with

robustness against a variety of attacks such as filtering but do not always address robustness to attacks mounted by a coalition of users with the same content that contains different marks. These attacks, which are known as collusion attacks, can provide a cost-effective approach to removing an identifying watermark.

Key words: Sperner family, Independence, Boolean operator, Truth table

## References

- [1]C-Y. Lin, M. Wu, Y-M. Lui, J.A. Bloom, M.L. Miller, and I.J. Cox, "Rotation, scale, and translation resilient public watermarking for images," *IEEE Trans. Image Processing*, vol. 10, pp. 767–782, May 2001.
- [2] J. Lubin, J. Bloom, and H. Cheng, "Robust, content-dependent, highfidelity watermark for tracking in digital cinema," *Security and Watermarking of Multimedia Contents V, Proc. SPIE*, vol. 5020, pp. 536–545, Jan. 2003.
- [3]W. Trappe, M. Wu, Z.J. Wang, and K.J.R. Liu, "Anti-collusion fingerprinting for multimedia," *IEEE Trans. Signal Processing*, vol. 51, pp. 1069–1087, Apr. 2003.
- [4]M. Wu and B. Liu, "Data hiding in image and video: Part-I—Fundamental issues and solutions," *IEEE Trans. Image Processing*, vol. 12, pp. 685–695, June 2003.