On a conjecture on the order of cages with a given girth pair

Camino Balbuena
(joint work with Julián Salas)
Departament de Matemàtica Aplicada III
Universitat Politècnica de Catalunya
Campus Nord, Edifici C2, C/ Jordi Girona 1 i 3
E-08034 Barcelona, Spain

Abstract A $(k; g, h)$-graph is a k-regular graph of girth pair (g, h) where g is the girth of the graph, h is the length of a smallest cycle of different parity than g and $g < h$. A $(k; g, h)$-cage is a $(k; g, h)$-graph with the least possible number of vertices denoted by $n(k; g, h)$. In this talk we prove that $n(k; g, h) \leq n(k, h)$ for all $(k; g, h)$-cages when g is odd, and for for g even and h sufficiently large provided that a bipartite (k, g)-cage exists. This conjecture was posed by Harary and Kóvacs in [2]. Also we include some comment about the last obtained upper bounds on the order of $(k; g, h)$-cages for $g = 6, 8, 12$ [1].

Bibliography

[1] C. Balbuena, J. Salas: Lower and upper bounds for the order of girth pair cages from Moore graphs, *Discrete Math.*, **321** (2014), 68–75.

[2] F. Harary, P. Kovács: Regular graphs with given girth pair, *J. Graph Theory*, **7** (1983), 219–218.