Small bi-regular graphs of even girth

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A graph of girth g that contains vertices of degrees r and m is called a bi-regular graph and denoted by $(\{r, m\}, g)$-graph. In analogy with the *Cage Problem*, we seek the smallest $(\{r, m\}, g)$-graphs for given parameters $2 \leq r < m$, $g \geq 3$, called $(\{r, m\}, g)$-cages.

Recently, Jajcay and Exoo, constructed an infinite family of $(\{r, m\}, g)$-cages for m much larger than r and odd girth g whose orders match a well-known lower bound given by Downs, Gould, Mitchem and Saba in 1981. Also they proved that a generalization of this result to bi-regular cages of even girth is impossible, because if the girth is even the bi-regular cages never match this lower bound.

In 2003, Yang and Liang, given a lower bound of the order of $(\{r, m\}, 6)$-cages and they constructed families of graphs that match this lower bound. In 2008, Araujo-Pardo, Balbuena, García Vázquez, Marcote and Valenzuela showed lower bounds for any even girth, and constructed more families of graphs that match the lower bound for $(\{r, m\}, 6)$-cages.

In this work, we summarize and improve some of these lower bounds for the orders of bi-regular cages of even girth and present a generalization of the odd girth construction to even girth that provides us with a new general upper bound on the order of graphs with girths of the form $g = 2t$, t odd. This construction gives us infinitely many $(\{r, m\}, 6)$-cages with sufficiently large m. We also determine a $(\{3, 4\}, 10)$-cage of order 82.

References:

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