On Graphs with Excess or Defect 2

Frederik Garbe
FU Berlin

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Abstract

The Moore bound $m(d,k) = 1 + d\sum_{i=0}^{k-1}(d-1)^i$ is a lower bound for the number of vertices of a graph by given girth $g = 2k+1$ and minimal degree d. Hoffmann and Singleton [5], Bannai and Ito [1], Damerell [4] showed that graphs with $d > 2$ tight to this bound can only exist for girth 5 and degree $3, 7, 57$. The difference to the Moore bound by given girth is called the excess of a graph. In the case of girth 5 Brown showed in [3] that there are no graphs with excess 1 and Bannai and Ito showed in [2] that for $g \geq 7$ there are also no graphs with excess 1. We generalize the result of Kovács [6] that, under special conditions for the degree d, there are no graphs with excess 2 and girth 5 and prove the new result that an excess-2-graph with odd degree and girth 9 cannot exist. In this proof we discover a link to certain elliptic curves. Furthermore we prove the non-existence of graphs with excess 2 for higher girth and special valencies under certain congruence conditions. The results can be modified to fit the degree/diameter problem and lead to similar statements for graphs with defect 2.

References

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