Subgraphs of cages

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Infinite families of cages are not known except if the girth is 6, 8 or 12 (aside from the trivial cases). In these cases, (k, g) Moore-graphs exist whenever k is the successor of a prime power; namely, these are the incidence graphs of generalized polygons. This fact is often exploited by providing constructions of small (k, g)-graphs based on these families. In this talk we will focus mainly on the regular subgraphs of the incidence graphs of generalized polygons from a finite geometrical viewpoint with an emphasis on projective planes.

First we investigate induced regular subgraphs of generalized polygons. In general, a perfect t-fold dominating set (t-PDS) in a graph G is a proper subset D of the vertices such that all vertices of G not in D have exactly t neighbors in D. Clearly, if G is k-regular, then the complement of a t-PDS induces a $(k-t)$-regular subgraph of G. Thus, to obtain a small $(k-t)$-regular subgraph for a fixed t, we should find a large t-PDS. In a generalized polygon a t-PDS consists of a point set $\mathcal{P}_0$ and a line set $\mathcal{L}_0$ such that each line not in $\mathcal{L}_0$ is incident with exactly t points of $\mathcal{P}_0$, and each point not in $\mathcal{P}_0$ is incident with exactly t lines of $\mathcal{L}_0$. Such a pair $(\mathcal{P}_0, \mathcal{L}_0)$ is also called a t-good structure.

In the talk we will describe all t-good structures in (equivalently, all induced $(q+1-t)$-regular subgraphs of the incidence graph of) desarguesian projective planes, provided that t is small enough compared to the order q of the plane and the characteristic of the coordinatizing field. We will also consider regular non-induced subgraphs, which are much trickier, and in some cases yield slightly better results. We will show some new constructions as well.