

Diameter 2 Cayley Graphs of Dihedral Groups

Grahame Erskine
Department of Mathematics and Statistics
The Open University
Milton Keynes, UK
Grahame.Erskine@open.ac.uk

Abstract

For a general graph of diameter 2 and maximum degree d , the largest possible order is asymptotically d^2 . For Cayley graphs it is known that there is a relatively sparse family of groups (affine groups over finite fields of characteristic 2) for which this asymptotic limit can be attained. For more elementary families of groups, less is known. For example, for abelian groups the theoretical maximum asymptotic limit is $d^2/2$ but no construction is known which achieves this bound.

In this talk we consider the degree-diameter problem for Cayley graphs of dihedral groups with diameter 2 and degree d . We show a construction based on Galois fields which has asymptotic limit $d^2/2$, and a counting argument which shows that this is in fact asymptotically best possible. Thus we completely determine the asymptotic behaviour of this class of graphs.