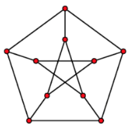
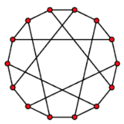
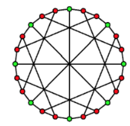
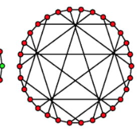
IWONT 2014





The 6th International Workshop on Optimal Network Topologies





June 30 - July 4, 2014, Bratislava, Slovakia

Conference Proceedings
List of Abstracts

IWONT 2014

June 30 - July 4, 2014, Bratislava, Slovakia

http://euler.doa.fmph.uniba.sk/IWONT2014.html

Editors: Prof. RNDr. Robert Jajcay, PhD.
Doc. RNDr. Jaroslav Guričan, PhD.
RNDr. Tatiana Jajcayová, PhD.

UNIVERZITA KOMENSKÉHO V BRATISLAVE
FAKULTA MATEMATIKY, FYZIKY A INFORMATIKY

Katedra algebry, geometrie a didaktiky matematiky

The 6-th International Conference on Optimal Network Topologies took place at the College of Mathematics, Physics and Computer Science of the Comenius University, Bratislava, between the days of June 30. and July 4.

Traditionally, the aim of the workshop is to nurture collaboration, jump start new projects, and serve as a stage for presenting the most recent results in the areas of

- Degree/Diameter Problem
- Connectivity
- Cycles and Factors in Graphs
- Construction Techniques for Large Graphs and Digraphs
- Structural Properties of Large Graphs and Digraphs Cages
- Extremal Graphs

This year's workshop took the usual form of four and a half days of invited and contributed talks with a half day break/trip on Wednesday. The conference was attended by over 40 active participants from 15 countries, and this year's keynote invited speakers included the following leading experts in the areas of interest of the conference:

- Edy Tri Baskoro
- Geoff Exoo
- Tamás Héger
- Domenico Labbate
- Felix Lazebnik
- Wendy Myrvold
- Vasyl Ustimenko
- Sanming Zhou

The Organizing Committee of IWONT 2014 consisted of: Robert Jajcay, Jozef Širáň, Tatiana Jajcayová, Katarína Hriňáková, and Katarína Tureková.

We would like to take one more opportunity to thank all the participants and express a hope that all of them enjoyed their stay in Bratislava and found the conference inspiring and productive.

Tatiana Jajcayová

For the organizing committee,
Prof. Robert Jajcay, PhD., Tatiana Jajcayová, PhD.
College of Mathematics, Physics and Computer Science
Comenius University

The following is the list of abstracts for all the talks presented at the conference ordered alphabetically with respect to the last name of the presenter.

A construction of Cayley graphs of diameter two and any degree with order $\frac{d^2}{2}$

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The number of vertices of a graph of diameter two and maximum degree d is at most d^2+1. This number is the Moore bound for diameter two. The order of largest Cayley graphs of diameter two and degree d is denoted by $C(d,2)$. The only known construction of Cayley graphs of diameter 2 valid for all degrees d gives $C(d,2) > \frac{1}{4}d^2+d$. However, there is a construction yielding Cayley graphs of diameter 2, degree d and order $d^2 - O(d^{\frac{3}{2}})$ for an infinite set of degrees d of a special type [1]. We present a construction giving $C(d,2) \geq \frac{1}{2}d^2 - k$ for d even and of order $C(d,2)\frac{1}{2}(d^2+d) - k$ for d odd, $0 \leq k \leq 8$. In addition, we show that, in asymptotic sense, the most of record Cayley graphs of diameter two are obtained by our construction.

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Research was supported by the VEGA Research Grant No. 1/0811/14.

Small bi-regular graphs of even girth

Gabriela Araujo-Pardo[1], Geoffrey Exoo[2], Robert Jajcay[3] *

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A graph of girth g that contains vertices of degrees r and m is called a bi-regular graph and denoted by $(\{r,m\},g)$-graph. In analogy with the *Cage Problem*, we seek the smallest $(\{r,m\},g)$-graphs for given parameters $2 \leq r < m$, $g \geq 3$, called $(\{r,m\},g)$-cages.

Recently, Jajcay and Exoo, constructed an infinite family of $(\{r,m\},g)$-cages for m much larger than r and odd girth g whose orders match a well-known lower bound given by Downs, Gould, Mitchem and Saba in 1981. Also they proved that a generalization of this result to bi-regular cages of even girth is impossible, because if the girth is even the bi-regular cages never match this lower bound.

In 2003, Yang and Liang, given a lower bound of the order of $(\{r,m\},6)$-cages and they constructed families of graphs that match this lower bound. In 2008, Araujo-Pardo, Balbuena, García Vázquez, Marcote and Valenzuela showed lower bounds for any even girth, and constructed more families of graphs that match the lower bound for $(\{r,m\},6)$-cages.

In this work, we summarize and improve some of these lower bounds for the orders of bi-regular cages of even girth and present a generalization of the odd girth construction to even girth that provides us with a new general upper bound on the order of graphs with girths of the form $g = 2t$, t odd. This construction gives us infinitely many $(\{r,m\},6)$-cages with sufficiently large m. We also determine a $(\{3,4\},10)$-cage of order 82.

References:

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On a conjecture on the order of cages with a given girth pair

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(joint work with Julián Salas)
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Abstract A $(k; g, h)$-graph is a k-regular graph of girth pair (g, h) where g is the girth of the graph, h is the length of a smallest cycle of different parity than g and $g < h$. A $(k; g, h)$-cage is a $(k; g, h)$-graph with the least possible number of vertices denoted by $n(k; g, h)$. In this talk we prove that $n(k; g, h) \leq n(k, h)$ for all $(k; g, h)$-cages when g is odd, and for for g even and h sufficiently large provided that a bipartite (k, g)-cage exists. This conjecture was posed by Harary and Kóvacs in [2]. Also we include some comment about the last obtained upper bounds on the order of $(k; g, h)$-cages for $g = 6, 8, 12$ [1].

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On graph Ramsey numbers for wheels and union of graphs

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2000 Mathematics Subject Classification. : 05C55, 05D10

Ramsey theory was initially studied in the context of the problem of finding a regular procedure to determine the consistency of any given logical formula (1928). This became famous after Paul Erdös and George Szekeres (1935) applied it in graph theory.

The research on finding the exact value of classical Ramsey numbers $R(m, n)$ has received a lot of attention. However, the results are still far from satisfactory. On the other hand, graph Ramsey theory as one of its generalizations has grown enormously in the last four decades to become presently one of the most active areas in Ramsey theory.

Let G and H be two graphs. Basically, the *Ramsey number* $R(G, H)$ is defined as the smallest integer N such that any 2-colouring (red or blue) on the edges of K_N yields either a red subgraph G or a blue subgraph H. The determination of Ramsey numbers $R(G, H)$ has been studied for various combinations of graphs G and H. In this talk, we shall give a survey on the determination of Ramsey numbers $R(G, H)$ if either G or H is a wheel. We also discuss the Ramsey numbers $R(G, H)$ if either G or H is a union of graphs.

Applying Sperner antichain to digital fingerprint detection

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Abstract

Sperner family, formally an antichain in the inclusion lattice over the power set of a universal set X, is also called an independent system. The independence is defined as the non-containing-ship between every pair of members. In other words, the dependence is defined as the existence of a containing-ship for some pairs. This is a relation between two members. In contrast, the dependence in linear algebra is defined by the relation between one member and one group (many members). We therefore ask if this relational difference for the Sperner family is appropriated. Is this dependence relation defined for the Sperner member too strict?

An independent system (the Sperner family) is interpreted by us to have no redundant member in the family. Redundant member is clearly understood by words is a member, who does make any difference for the family if he exists or not. By this interpretation, the dependence relation is built upon the redundant member with the rest of the family. However, we shall prove this relation finally becomes to the personal relation between two. To check if there is a difference made by the suspicious redundant member, the originally "static" member needs to be regarded as an operator to have the ability to influence. One simple arrangement is to interpret the family (union of members) as a Boolean operator composed a sum (logic OR) of products (logic AND). For example, the family {[11000], [10100], [01100]} is regarded as the operator ab + ac + bc.

Under the above operator interpretation (interpretation 2, relative to the interpretation 1 about redundancy), two proofs are developed for the goal that the interpretation 1 leads to the original definition of containing-ship between two members used in the Sperner family. One is proved by truth table method. We also visualize this method by designing a full-pattern (the all possible inputs in the truth table) image to be processed by the family operator and the family minus one operator (ab + ac + bc vs. ab + ac for example) to check the difference. The second proof is by transforming the logic operation to propositional calculus.

Using this interpretation, the Sperner family can be easily extended to many applications for a compact purpose. In order to control the redistribution of content, digital fingerprinting is used to trace the consumers who use their content for unintended purposes [1-4]. These fingerprints can be embedded in multimedia content through a variety of watermarking techniques. Conventional watermarking techniques are concerned with

Communicability in Cubic Generalized Moore Graphs

Francesc Comellas
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Generalized Moore graphs (GMG) are regular graphs which attain the generalized Moore bound, a bound related to the Moore bound for the degree-diameter problem. A GMG is a graph that, for a given order and degree, has minimal average distance. Thus, for a GMG of degree Δ the number of vertices at each distance $1, 2, 3, \ldots$ from any vertex is $\Delta, \Delta(\Delta-1), \Delta(\Delta-1)^2, \ldots$ with the last level not necessarily filled up. The girth g and diameter D of a GMG satisfy $g \geq 2D-1$.

Generalized Moore graphs were introduced by Cerf, Cowan, Mullin and Stanton in a series of papers published in the 70's, see for example [1]. Further work, for the $\Delta = 3$ case, was published by McKay and Stanton [3]. Surprisingly not much research has been done on this topic since then and relevant questions, like if there are infinitely many GMG for each degree, are still open.

In my talk I will survey known results for cubic GMG and present work in progress with respect the communicability [2] (a measure of closed walks starting and ending at a node) and other related properties for non isomorphic cubic generalized Moore graphs with the same order.

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The degree/diameter problem in maximal planar bipartite graphs *

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Abstract

The (Δ, D) (degree/diameter) problem consists of finding the largest possible number of vertices n among all the graphs with maximum degree Δ and diameter D. We consider the (Δ, D) problem for maximal planar bipartite graphs, that are simple planar graphs in which every face is a quadrangle. We obtain that for the $(\Delta, 2)$ problem, the number of vertices is $n = \Delta + 2$; and for the $(\Delta, 3)$ problem, $n = 3\Delta - 1$ if Δ is odd and $n = 3\Delta - 2$ if Δ is even. Then, we study the general case (Δ, D) and obtain that an upper bound on n is approximately $3(2D + 1)(\Delta - 2)^{\lfloor D/2\rfloor}$, and another one is $C(\Delta - 2)^{\lfloor D/2\rfloor}$ if $\Delta \geq D$ and C is a sufficiently large constant. Our upper bounds improve for our kind of graphs the one given by Fellows, Hell and Seyffarth for general planar graphs. We also give a lower bound on n for maximal planar bipartite graphs, which is approximately $(\Delta - 2)^k$ if $D = 2k$, and $3(\Delta - 3)^k$ if $D = 2k + 1$, for Δ and D sufficiently large in both cases.

*Research of C. Dalfó supported by the Ministerio de Educación y Ciencia and the European Regional Development Fund under project MTM2011-28800-C02-01, and by the Catalan Research Council under project 2009SGR1387. Research of C. Huemer partially supported by projects MEC MTM2012-30951 and Gen. Cat. DGR 2009SGR1040 and ESF EUROCORES programme EuroGIGA, CRP ComPoSe: grant EUI-EURC-2011-4306. Research of J. Salas supported by the Ministry of Education and Science, Spain, and the European Regional Development Fund (ERDF) under project MTM2011-28800-C02-02.

Diameter 2 Cayley Graphs of Dihedral Groups

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Abstract

For a general graph of diameter 2 and maximum degree d, the largest possible order is asymptotically d^2. For Cayley graphs it is known that there is a relatively sparse family of groups (affine groups over finite fields of characteristic 2) for which this asymptotic limit can be attained. For more elementary families of groups, less is known. For example, for abelian groups the theoretical maximum asymptotic limit is $d^2/2$ but no construction is known which achieves this bound.

In this talk we consider the degree-diameter problem for Cayley graphs of dihedral groups with diameter 2 and degree d. We show a construction based on Galois fields which has asymptotic limit $d^2/2$, and a counting argument which shows that this is in fact asymptotically best possible. Thus we completely determine the asymptotic behaviour of this class of graphs.

Computing Cages: A Survey of Computational Methods for the Cage Problem

Geoffrey Exoo
Indiana State University

July 1, 2014

Abstract

The cage problem has provided a variety of entertaining computational problems. In this talk we survey the state of the art from the programmer's perspective. Some general computer methods for constructing graphs with specified properties will be outlined. Findings that result from applying these methods to the cage and degree/diameter problems will be discussed.

A few new lower bounds for specific instances of the problem will be given. One of these *almost* leads to a new cage. In this case, we describe the missing pieces of an argument that might lead to the determination of the cage.

Finally, we discuss a problem in which nobody other than the speaker has ever expressed an interest: what is the maximum girth of a cubic graph that can be constructed on a computer?

A Spectral Characterization of Strongly Distance-Regular Graphs with Diameter Four.

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Abstract

A graph G with $d+1$ distinct eigenvalues is called strongly distance-regular if G itself is distance-regular, and its distance-d graph G_d is strongly-regular. In this talk we discuss the case of diameter $d=4$, and present a new spectral characterization of those distance-regular graphs with such a diameter which are strongly distance-regular.

Keywords: Distance-regular graph; Strongly distance-regular graph; Spectrum.

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Why chemists care about graph theory

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Graph theory has had close links with chemistry for at least a century and a half, from the early preoccupation with structural enumeration, through quantum mechanical models of electronic structure, to systematic nomenclature, study of the links between structure and properties of molecules, and the advent of 'combinatorial chemistry'. This talk starts with a personal perspective on what makes chemistry an ideal field for the application of graph theory, and goes on to describe some ideas in the burgeoning field of *molecular conduction*, which give yet another reason for chemists to 'care about graph theory'.

The talk includes work done in collaboration with Martha Borg, Rasthy De Los Reyes, Wendy Myrvold, Barry Pickup, Tomaz Pisanski, Irene Sciriha, and Tsanka Todorova.

Decompositions of complete bipartite graphs into prisms

Dalibor Froncek, University of Minnesota Duluth

A *generalized prism*, or more specifically an $(0, j)$-prism of order $2n$ (where n is even) is a cubic graph consisting of two cycles $u_0, u_1, \ldots, u_{n-1}$ and $v_0, v_1, \ldots, v_{n-1}$ joined by two sets of spokes, namely $u_1v_1, u_3v_3, \ldots, u_{n-1}v_{n-1}$ and $u_0v_j, u_2v_{j+2}, \ldots, u_{n-2}v_{j-2}$.

The question of factorization of complete bipartite graphs into $(0, j)$-prisms was completely settled by the author and S. Cichacz. Some partial results on decompositions of complete bipartite graphs and complete graphs have also been obtained by them and P. Kovar and S. Dib, respectively. The problem of decomposition of complete graphs into prisms of order 12 and 16 was completely solved by the author with S. Cichacz and M. Meszka and presented at IWONT 2012.

We will present a complete solution for the decomposition of complete bipartite graphs into $(0, 0)$-prisms (that is, the usual prisms).

Keywords: Graph decomposition, cubic graph, generalized prism

On Graphs with Excess or Defect 2

Frederik Garbe
FU Berlin

June 25, 2014

Abstract

The Moore bound $m(d,k) = 1 + d\sum_{i=0}^{k-1}(d-1)^i$ is a lower bound for the number of vertices of a graph by given girth $g = 2k+1$ and minimal degree d. Hoffmann and Singleton [5], Bannai and Ito [1], Damerell [4] showed that graphs with $d > 2$ tight to this bound can only exist for girth 5 and degree $3, 7, 57$. The difference to the Moore bound by given girth is called the excess of a graph. In the case of girth 5 Brown showed in [3] that there are no graphs with excess 1 and Bannai and Ito showed in [2] that for $g \geq 7$ there are also no graphs with excess 1. We generalize the result of Kovács [6] that, under special conditions for the degree d, there are no graphs with excess 2 and girth 5 and prove the new result that an excess-2-graph with odd degree and girth 9 cannot exist. In this proof we discover a link to certain elliptic curves. Furthermore we prove the non-existence of graphs with excess 2 for higher girth and special valencies under certain congruence conditions. The results can be modified to fit the degree/diameter problem and lead to similar statements for graphs with defect 2.

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Subgraphs of cages

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Infinite families of cages are not known except if the girth is 6, 8 or 12 (aside from the trivial cases). In these cases, (k, g) Moore-graphs exist whenever k is the successor of a prime power; namely, these are the incidence graphs of generalized polygons. This fact is often exploited by providing constructions of small (k, g)-graphs based on these families. In this talk we will focus mainly on the regular subgraphs of the incidence graphs of generalized polygons from a finite geometrical viewpoint with an emphasis on projective planes.

First we investigate induced regular subgraphs of generalized polygons. In general, a perfect t-fold dominating set (t-PDS) in a graph G is a proper subset D of the vertices such that all vertices of G not in D have exactly t neighbors in D. Clearly, if G is k-regular, then the complement of a t-PDS induces a $(k-t)$-regular subgraph of G. Thus, to obtain a small $(k-t)$-regular subgraph for a fixed t, we should find a large t-PDS. In a generalized polygon a t-PDS consists of a point set $\mathcal{P}_0$ and a line set $\mathcal{L}_0$ such that each line not in $\mathcal{L}_0$ is incident with exactly t points of $\mathcal{P}_0$, and each point not in $\mathcal{P}_0$ is incident with exactly t lines of $\mathcal{L}_0$. Such a pair $(\mathcal{P}_0, \mathcal{L}_0)$ is also called a t-good structure.

In the talk we will describe all t-good structures in (equivalently, all induced $(q + 1 - t)$-regular subgraphs of the incidence graph of) desarguesian projective planes, provided that t is small enough compared to the order q of the plane and the characteristic of the coordinatizing field. We will also consider regular non-induced subgraphs, which are much trickier, and in some cases yield slightly better results. We will show some new constructions as well.

On the Wiener index for iterated line graphs of trees

Martin Knor

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Joint work with Riste Škrekovski, Primož Potočnik and Martin Mačaj

Let G be a graph. The sum of all distances in G is called the Wiener index of G and it is denoted by $W(G)$. The i-iterated line graph of G, $L^i(G)$, is $L^i(G) = L(L^{i-1}(G))$, where L is the line-graph operator and $L^0(G) = G$. Let T denote a tree. It is known that $W(L(T)) \neq W(T)$, while $W(L^2(T)) = W(T)$ has infinitely many solutions. Dobrynin and Melnikov conjectured that $W(L^i(T)) = W(T)$ has no solution if $i \geq 3$. We disproved this conjecture and we characterized all i's and T's, $i \geq 3$, satisfying $W(L^i(T)) = W(T)$.

Speakers: Kristína Kováčiková
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Co-author: Martin Mačaj
Title: The numbers of induced subgraphs in strongly regular graphs

Let us fix a graph Γ. By P_G we denote the number of occurrences of graph G as an induced subgraph in Γ. Clearly, the values P_{K_1}, P_{K_2} and $P_{\overline{K}_2}$ represent the numbers of vertices, edges and non-edges in Γ, respectively.

A k-regular graph Γ of order n, where the number of common neighbours of any two vertices in Γ depends only on whether they are adjacent or not, is called a strongly regular graph ($SRG(n, k, \lambda, \mu)$). In this case it is known that the value P_G of any graph G on at most three vertices is determined uniquely by parameters of SRG. Unfortunately, with G spanning more than 3 vertices, this nice property is no longer satisfied. An example of such behavior are two non-isomorphic $SRGs$ with parameter set $(16, 6, 2, 2)$ and different values of P_{K_4}.

We study how the values of P_G for all the graphs on t vertices interact. For triangle-free SRG we show that P_G is determined by n, k, λ and μ for any G on at most five vertices. When G is a graph on six vertices, P_G depends also on the value $P_{K_{3,3}}$.

For putative Moore graph with parameters $(3250, 57, 0, 1)$, P_G is determined uniquely for any graph G on up to 9 vertices. For all graphs on 10 vertices the values P_G are dependent only on the number of occurrences of Petersen graph in this SRG.

Techniques for Constructing Small Regular Graphs of Given Girth and Related Topics

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The *Cage Problem* asks for the construction of regular simple graphs with given degree and girth and minimum order. A (k, g)*–graph* is a k–regular graph of girth g. Sachs proved in 1963 that (k, g)–graphs exists for each $k \geq 3$ and $g \geq 5$. *Moore's bound* is obtained when counting the minimum number of vertices necessary to construct a (k, g)–graph. A (k, g)–graph whose order attains Moore's bound is, by definition, also a *Moore graph*. It is well known that the Moore graphs exist for girth 5 and $k = 2, 3, 7$ and maybe 57 and girth $6, 8$ or 12 and they are incidence graphs of finite projective planes, generalized quadrangles or generalized hexagons, respectively. Moreover Hoffman, Singleton, Feit, Higman, Damerell, Bannai and Ito proved in the 60–70's that there are no further Moore graphs.

Thus, it is natural to approach the more general problem of determining the minimum order of (k, g)–graphs. We denote this minimum value by $n(k, g)$ and a graph attaining this minimum value is said to be a (k, g)*–cage*. Hence, in most cases the number of vertices in a (k, g)–cage is strictly greater than Moore's bound. Several authors are trying to construct (k, g)–cages, or at least smaller (k, g)–graphs than previously known ones.

In this talk, we will describe several techniques (algebraical, geometrical and purely combinatorial) that we used to construct small regular graphs of girth 5, 6, 7 and 8 as well as to solve some related problems such as the existence of symmetric configurations and the search for C_4–free graphs of large size. In particular, we will point out how these techniques are related and how they are helpful in solving the above mentioned problems. Moreover, we will present new and recent results obtained for small regular (k, g)–graph of girth 7 and 8 and for biregular $(\{r, m\}, g)$–graphs of girth 5. Finally, we will present some possible further developments of this topic.

ON WENGER GRAPHS

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Abstract. Let q be a prime power, and let $\mathbb{F}_q$ be the field of q elements. For any positive integer n, Wenger graph $W_n(q)$ is defined as follows: it is a bipartite graph with the vertex partitions being two copies of the $(n+1)$-dimensional vector space $\mathbb{F}_q^{n+1}$, and two vertices $(p) = (p_1, \dots, p_{n+1})$ and $[l] = [l_1, \dots, l_{n+1}]$ being adjacent if $p_i + l_i = p_1 {l_1}^{i-1}$, for $i = 2, 3, \dots, n+1$. In this talk we will survey properties of this interesting family of graphs, present several recent results, and mention some related open problems.

The degree-diameter problem for circulant graphs of degree 8 and 9

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Abstract

This talk considers the degree-diameter problem for undirected circulant graphs. The focus is on extremal graphs of given (small) degree and arbitrary diameter. The published literature only covers graphs of up to degree 7. The approach used to establish the results for degree 6 and 7 has been extended successfully to degree 8 and 9. Candidate graphs are defined as functions of the diameter for both degree 8 and degree 9. They have been proven to be extremal for small diameters. They establish new lower bounds for all greater diameters, and are conjectured to be extremal. Finally some conjectures are made about solutions and upper bounds for circulant graphs of higher degree.

The non existence of a Mixed Moore graph of order 486

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Abstract

Mixed graphs of order n such that for any pair of vertices there is a unique trail of length at most k between them are known as mixed Moore graphs. These extremal graphs may only exist for diameter $k = 2$ and some (infinitely many) values of n. In this talk we characterize mixed Moore graphs of directed degree one. In particular, we prove the non-existence of a mixed Moore graph of order 486 which is equivalent to saying that a directed strongly regular graph with parameters $(486, 22, 1, 0, 21)$ does not exist.

What to do with the missing Moore graph?

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Abstract

The existence of a regular 57-valent graph with diameter 2 and girth 5 is one of the most famous open problems in graph theory. However, due to the nature of the problem, the number of published papers devoted to this problem is extremely small. As a consequence, each person interested in the problem has to start from the beginning and the space for sharing ideas is limited. What can we, as a community, do to deal with this situation?

Radial Moore Graphs for Every Diameter

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The *degree* of a vertex is the number of its adjacent edges; the *diameter* of a graph is the largest distance between any two vertices. The *degree/diameter problem* asks, for given maximum degree and given diameter, what is the largest number of vertices that a graph can have?

A natural upper bound for the degree/diameter problem is the so-called Moore bound.

A *radial Moore graph* is a graph of maximum degree d, radius k and diameter at most k+1, while the number of vertices is equal to the Moore bound M(d,k).

It has been an open problem for more than a decade to find if a radial Moore graph exists for every value of k. In this talk we will present some new results concerning radial Moore graphs for any given radius. The talk will conclude with some further open problems.

Exploring connections between chemistry, computer science, and graph theory

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The goal of this talk is to illustrate interconnections between chemistry and interesting theoretical and algorithmic questions in graph theory.

We will consider several classes of molecules that can be represented by planar graphs that have maximum degree three. A *fullerene* is an all carbon molecule that corresponds to a 3-regular planar graph with face sizes five or six. *Fusenes* are hydrocarbon molecules that correspond to simple planar 2-connected graphs embedded in the plane such that all internal faces are hexagons, all vertices not on the external face have degree 3 and vertices on the external face have degree 2 or 3. *Benzenoids* have similar structure but can contain holes.

Various graph theory concepts are of chemical relevance. A subset S of the vertices of a graph forms an *independent set* if the vertices of S are pairwise non-adjacent. Independent sets model addition possibilities for reactions with bulky addends. Given a perfect matching of a graph, a *benzenoid hexagon* is a hexagon which contains three matching edges. One simple chemical model for stability of benzenoid molecules uses the *Fries number* (the maximum number of benzenoid hexagons over all the perfect matchings of the molecular graph). Another model uses the *Clar number* (the maximum number of independent benzenoid hexagons over all perfect matchings). A *conjugated circuit* of a graph G is a cycle C such that $G - C$ has a perfect matching. Several simplified models for currents are based on enumerating contributions from conjugated circuits.

This talk will summarize the work we have done so far on independent sets, the Clar and Fries numbers, and currents in molecules. The work discussed has been done in collaboration with Patrick Fowler, a chemist from the University of Sheffield, and several students at the University of Victoria: William H. Bird, Matthew J. Imrie, and Sean Daugherty (currently at Metron Inc.).

Enumeration Of $(3, g)$ Hamiltonian bipartite graphs upto $2^{3g/4}$ vertices

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Abstract

We present a range of $(3, g)$ Hamiltonian bipartite graphs for a range for vertices for even values of girth g satisfying $6 \leq g \leq 16$ and present a new methodology to analyze the trivalent cage problem for even girth. Our lists of $(3, g)$ Hamiltonian bipartite graphs have been found to be significantly more dense than other known lists of $(3, g)$ graphs, and hence allow confirmation of the $(3, g)$ upper bounds. This would be even more useful for the Cage Problem if these general computational approaches could be made to work for higher values of girth.
We pose a problem of enumerating $(3, g)$ Hamiltonian bipartite graphs upto $2^{3g/4}$ vertices that is motivated by the open problem, "Finding an infinite family of trivalent graphs with large girth g and order 2^{cg} for $c < 3/4$."
We also introduce $(3, g)$ sub-problems as follows. We decompose the problem of finding the smallest $(3, g)$ Hamiltonian bipartite graph to sub-problems of for finding $(3, g)$ Hamiltonian bipartite graph with symmetry factor b having the minimum number of vertices. Symmetry factor is a parameter that reflects the extent of rotational symmetry.
The enumeration of $(3, g)$ Hamiltonian bipartite graphs with symmetry factor b upto $2^{3g/4}$ vertices corresponds to the sub-problem for finding $(3, g)$ Hamiltonian bipartite graph with symmetry factor b having the minimum number of vertices.
We consider the enumeration of $(3, g)$ Hamiltonian bipartite graphs upto $2^{3g/4}$ to be exhaustive if all even vertices less than or equal to $2^{3g/4}$ that have a $(3, g)$ Hamiltonian bipartite graph are listed, with proof for non-existence for vertices not listed, and at least one $(3, g)$ Hamiltonian bipartite graph for each of the vertices listed. If list of vertices for which $(3, g)$ Hamiltonian bipartite graphs upto $2^{3g/4}$ is not complete, then it is a partial list. We consider two cases of partial enumeration, where results on existence $(3, g)$ Hamiltonian bipartite graph for some vertices listed in specified range are inconclusive, but upper bound for $(3, g)$ for even girth g can be confirmed from the list of $(3, g)$ Hamiltonian bipartite graphs.
We show that $(3, 6)$ Hamiltonian bipartite graphs exist for all even vertices greater than equal to 14. We confirm that there exists only one Hamiltonian bipartite $(3, 6)$ cage by enumeration and isomorphism checking for 14 vertices, girth 6 and symmetry factor 7.
We show that $(3, 8)$ Hamiltonian bipartite graphs exist for all even vertices between 30 and 90, with the exception of 32, for which we show that a $(3, 8)$ Hamiltonian bipartite graph with 32 vertices does not exist. We resolve sub-problems for $(3, 8)$ Hamiltonian bipartite graphs and find the minimum number of vertices for $(3, 10)$

Mixed Cayley graphs of diameter two of order asymptotically approaching the Moore bound

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For a range of parameters Δ and d we present constructions of mixed Cayley graphs of 'undirected' degree Δ, 'directed' degree d and diameter 2 such that the ratio of their order and the quantity $(\Delta+d)^2+d+1$ (the mixed Moore bound for diameter 2) tends to 1 as $\Delta + d \to \infty$.

Metric Dimension for Smallest Regular Graphs of Given Degree and Diameter

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Abstract

A set of vertices S resolves a graph G if every vertex is uniquely determined by its vector of distances to the vertices in S. The metric dimension of G is the minimum cardinality of a resolving set of G.

Recently, Knor [6] gave a sharp lower bound on the number of vertices in a regular graph of given degree and diameter. Here we study the metric dimensions of graphs achieving such lower bound.

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Covering constructions in the degree-diameter and degree-girth problems revisited

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A considerable proportion of the largest currently known graphs of given degree and diameter, and the smallest known graphs of given degree and girth, have been or can be obtained as lifts of small base graphs with voltages in groups with a fairly simple structure. In the talk we will revisit such constructions and outline open problems in this area of research.

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Title: Graphs similar to strongly regular graphs

The degree/diameter problem is the problem of finding the largest possible graph with given diameter d and given maximum degree k. For graphs with diameter 2 the upper bound (Moore bound) is simplified to k^2+1. In 1980 Erdös, Fajtlowicz and Hoffman showed that, with the exception of the cycle of length 4, there does not exist any k-regular graph with diameter 2 and k^2 vertices (such graph has order one less than the Moore bound). Authors reduced this problem to solving the matrix equation

$$A^2 + A - (k-1)I = J + K,$$

where A is the adjacency matrix of the graph, I is the identity matrix, J is the all-ones matrix and K is the matrix of a suitable 1-factor.

Our aim is to solve the generalisation of the previous problem to one in which we replace Moore graphs with diameter 2 by strongly regular graphs. That is, we are looking for k-regular graphs on n vertices such that their adjacency matrix A satisfies the equation

$$A^2 + (c-a)A + (c-k)I = cJ + K.$$

We derive necessary conditions for parameters (n, k, a, c) analogous to the integral criterion for strongly regular graphs. In this process the systemic application of algebraic properties of the third power of adjacency matrix A^3 proves to be crucial. Finally we find the complete (infinite) list of parameters satisfying these necessary conditions. Existence of graphs with these parameters remains an open problem.

On graphs of large size without small cycles and commutative diagrams and their applications

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We define a cycle indicator of a vertex of a simple graph as a minimal cycle through the vertex. A cycle indicator of a graph is a maximum of cycle indicator of their vertices. The maximal size of the graph with a given cycle indicator is evaluated. This bound turns out to be sharp in difference with the Even Circuit Theorem by P. Erdős and its corollary for graphs of given girth. The sharpness is proven explicitly by a construction of the family of small world graphs with increasing cycle indicator, such that their magnitude is on a new bound.

Let us refer to a directed graph Γ as balanced directed graph if it is a graph without multiple arrows such that numbers of inputs and outputs are the same for every vertex.

The class of a balanced directed graphs is an extension of the class of simple graphs for which the concept of a girth can be naturally defined. We evaluate precisely the maximal size of balanced directed graph on v vertices of girth $> d$.

Concepts of a family of small world graphs and a family of graphs of large girth can be generalized on a class of balanced directed graphs.

We prove, that for each pair (K, S), where K is commutative ring and S be its multiplicatively closed subset without zero, there exists an infinite directed regular balanced graph $\Gamma_S(K)$ without commutative diagrams.

We will use well defined functor $(K, S) \to \Gamma_S(K)$ for the construction of families of graphs of large girth, graphs with large cycle indicator, small world graphs for which $\Gamma_S(K)$ will appear as well defined projective limit.

The brief survey of applications of $\Gamma_S(K)$ to Information Security will be given.

A family of mixed dense graphs of diameter 2

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Abstract

In this paper we give a construction of mixed dense graphs of diameter 2, undirected degree q, directed degree $\frac{q-1}{2}$, and order $2q^2$, when q is an odd prime power. Since the Moore bound for a mixed Moore graph with these parameters is equal to $\frac{9q^2-4q+3}{4}$, the defect is $(\frac{q-2}{2})^2 - \frac{1}{4}$.

In particular for $q = 5$ we construct a mixed graph of order 50, undirected degree 5 and directed degree 2. Since Bosák proved (in 1979) that there does not exist a mixed Moore graph with these values of degree and diameter, and since it is easy to see that a mixed graph of the same parameter values and with one vertex less than the Moore bound also does not exist, it turns out that our graph is the largest possible.

Key words. Mixed Moore graphs, diameter, tournament.

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Constructing Cayley graphs for efficient data transmission

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Abstract

It is well known that Cayley graphs play an important role in the design of interconnection networks due to many attractive properties they exhibit. In fact, a number of networks of both theoretical and practical importance are Cayley graphs. In the past more than two decades researchers proposed many families of Cayley graphs as models for interconnection networks. I will talk about some efforts in recent years towards constructing Cayley graphs that are efficient for data transmission measured by transmission time, broadcasting time and/or edge-congestion.