Fifth Workshop on

Constructions of Expanders and Extremal Graphs

List of Abstracts

Slovakia-Austria Bilateral Collaboration

SK-AT-23-0019
Smolenice and Faculty of Mathematics, Physics and Computer Science of the Comenius University
November 5 - 10, 2025

The following is the list of abstracts for talks presented at the workshop ordered chronologically as presented.

Wednesday, November 5, 14:30

Christopher Cashen, TU Wien

An application of a family of edge-girth-regular graphs to a problem about the coarse geometry of right-angled Coxeter groups

Abstract: Jajcay-Kiss-Miklavič (2018) defined edge-girth-regular (v, k, g, λ) graphs to be graphs with v vertices, valence k, girth g, and containing λ-many girth cycles through each edge. Araujo-Pardo and Leemmans (2022) constructed some families of edge-girth-regular graphs. One such family consists of the Levi graph of the biaffine plane over the finite field of order q, for each prime power $q > 2$. We will show that these graphs can be used to construct hyperbolic right-angled Coxeter groups whose boundaries have arbitrarily high conformal dimension, and that are different from previously known constructions of such groups.

Wednesday, November 5, 15:40

Pavol Kollár, Comenius University

Boundary matrices and their connection to de Bruijn graphs

Abstract: The methods with which we enummerate and count things can vary significantly based on the problem at hand. In this talk, we focus on applying various approaches to the following problem, which has already been explored in [1, 2]:

Definition 1. *Let $\mathcal{A}$ be a fixed alphabet of symbols and for fixed $k, l \in \mathbb{Z}^+$, let Z be a set of $k \times l$ matrices over $\mathcal{A}$. Then a **boundary matrix** (w.r.t. $\mathcal{A}, Z$) is any matrix over alphabet $\mathcal{A}$ which does not contain any matrix $Z_i \in Z$ as a contiguous submatrix.*

In this sense, *Boundary Matrices* are a two-dimensional generalisation of the antidictionary languages, which is a classical - solved - problem [3].

The problem we focus on is determining, for fixed size $m \times n$, the number of boundary matrices as a function $B(\mathcal{A}, Z, m, n)$, in addition to seeking relations between the values of this function. It is known that

Theorem 1. *For $m \in \mathbb{Z}^+$ fixed, there exists a linear recurrence relation in n for the function values of $B(\mathcal{A}, Z, m, n)$.*

However, it is not known if there exists a "two dimensional linear recurrence relation" for the values of B, which would be a much more powerful way of computing these values. This is one of our main focus points in this research project and as we will see, such a relation often does not exist.

Remark. Boundary matrices are an example of structures that are locally testable; a matrix is boundary, if non of its windows have the forbidden pattern(s). Related ideas

have been explored in the context of block gluing [4] and in measuring the entropy of ice [5].

Recently we discovered a close relationship between these matrices and the famous *de Bruijn graphs.*

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Reference

[1] Robert Jajcay, Enumeration of border matrices. *Matematicke obzory 31* (1989) 41–49.

[2] Robert Jajcay, Tatiana Jajcayova, Marian Opial, Enumeration of matrices with prohibited bounded sub-windows. *Conference Information Technologies – Applications and Theory 2018* (2018) 176–180.

[3] Alfred V. Aho, Margaret J. Corasick, Efficient string matching: an aid to bibliographic search, *Commun ACM.* (1975) 333–340.

[4] Silvère Gangloff, Mathieu Sablik, Quantified block gluing for multidimensional subshifts of finite type: aperiodicity and entropy. *Journal d'Analyse Mathématique* (2021) 21–118.

[5] Elliott H. Lieb, Exact Solution of the Problem of the Entropy of Two-Dimensional Ice. *Physical Review Letters* (1967) 692–694.

Thursday, November 6, 10:00

Markus Steenbock, University of Vienna

Product set growth for groups acting on trees

Abstract: We will be interested in finding lower bounds on the growth rate of free subsemigroups in groups acting on trees. We discuss a theorem of Safin on free groups that states that there is $a > 0$ such that for every finite subset U we have that $|U^3| > (a|U|)^2$. Under appropriate assumptions, we extend this estimate to groups acting on trees.

Friday, November 7, 9:30

Ján Pastorek, Comenius University

Partial automorphism inverse monoids and Maximal Asymmetric Depth of graphs

Abstract: Almost all graphs are asymmetric, possessing no non-trivial global automorphisms, yet all non-trivial graphs contain non-trivial local symmetries and structures which we study using isomorphisms between induced subgraphs, known as partial automorphisms. The set of all partial automorphisms, along with the operations of partial composition and partial inverse of partial maps, forms an inverse monoid, *partial automorphism inverse monoid*. In this talk, we highlight partial automorphism inverse monoids as a natural framework for understanding local symmetry.

We investigate the extent of these local symmetries on graphs through the measure of asymmetric depth of graphs defined through the rank of the largest non-trivial partial automorphism. We established a new, tight lower bound for the asymmetric depth of any simple graph Γ on n vertices. Any graph achieving this bound must be a strongly regular graph with parameters $(n, \frac{n-1}{2}, \frac{n-5}{4}, \frac{n-1}{4})$ also known as *conference graph*. We implemented a parallel algorithm for checking asymmetric depth on a high-performance cluster. Using this algorithm, we identified an asymmetric conference graph on 37 vertices that meets this bound, thereby proving its tightness. We also showed that it is one of the smallest possible graphs to meet this bound by checking all asymmetric conference graphs up to 37 vertices.

This is joint work with Tatiana Jajcayová

Keywords: partial automorphism, inverse monoid, asymmetric depth, conference graphs.

Monday, November 10, 11:30

Štefánia Glevitzká, Comenius University

Combinatorial aspects of the diameter/girth ratio problem

Abstract: In this talk, we address two problems related to the degree, girth and diameter of a graph. The first problem is the problem of finding an infinite family of k-regular graphs Γ_i of girth g_i and diameter d_i such that there exists a constant upper bound on $\frac{d_i}{g_i}$, for all i (we call this problem the *Diameter/Girth Ratio Problem*).

The second problem combines the well-known cage problem and the degree/diameter problem by considering graphs of fixed degree, girth, and diameter, the (k, g, D)-graphs. In case of these graphs, there exist both a lower and an upper bound on their orders, and so it makes sense to investigate the smallest and largest graphs for any given parameters (k, g, D) (assuming the existence of at least one such graph).

We present some simple combinatorial results concerning these graphs and explore the connections between the first and second problem, with the main observation being that any family of graphs of bounded diameter/girth ratio consists of (k, g_i, D_i) graphs satisfying the inequalities $\frac{d_i}{c} \leq g \leq 2d_i + 1$.

Monday, November 10, 13:30

Robert Jajcay, Comenius University

On the connections between Biggs groups, maps and polytopes

Abstract: Although the concept of *Biggs groups* has been introduced in the context of Cayley graphs of prescribed valency and girth, permutation groups generated by involutions pop up all over Mathematics. In our presentation we discuss the concept of the *monodromy group of a map* generated by 3 involutions, and then proceed to the *automorphism groups of polytopes* that can be (very loosely) viewed as generalizations of maps. The main aim is to use the monodromy groups and the automorphism groups of polytopes and their connections to Biggs groups in our investigation of Biggs groups.