What Is a Map? Multiple Representation

Thomas Tucker

"The difference between algebra and geometry? Well, with algebra it is sort of turning the crank, but with geometry you need an idea"
Solomon Lefschetz

"Civilization advances by extending the number of important operations which we can perform without thinking of them."
Alfred North Whitehead

April 13, 2021

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Your assignment Google "Median Gould" ,"McLuhan" "Principia Newton" , "Mecanique anal..." Start now.

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But how do you turn the crank? For example, put it on a computer?

Topological Map: Graph Embedding (cellular)

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Ringel started using face rotations for embeddings of K_n in the 1950s in his work on the Heawood Conjecture.
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Flags and flag graphs

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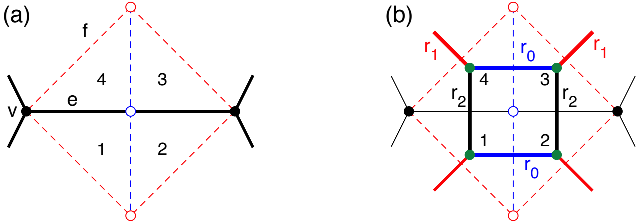

Figure: Four flags lying on an edge

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Can view as right action of a group G on the right cosets of a subgroup H of index $4n$.The group together with generators r_0, r_1, r_2 is the *monodromy* of the map.

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2. No symmetry: graph minors (Robertson-Seymour), genus of a graph, polynomials (Bollobas-Riordan)

Covering spaces: Going Down

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Example Suppose T is a connected graph.

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Theorem A covering induces a monomorphism $p* : \pi_1(X, x) \to \pi_1(Y, y)$ and for every subgroup of $\pi(T, y)$ there is a covering $p : X \to Y$ with $p * (\pi(X, x)$ that subgroup. Permutation representation.

Theorem(regular covering) If $p * (\pi(X, x)$ is normal in $\pi_1(X, x)$, then the quotient group G acts on X by "deck-transformations": homeomorphisms f such that $pf = p$. Then $p^{-1}(x)$ can be identified with G and action of deck-transformations is left multiplication by G.

For surfaces where X is closed, actually want **branched** coverings where there is a finite subset B of discrete points such that at each point of $p^{-1}(B)$ the covering is not locally one-to-one but instead looks like $z \to z^n$ in complex plane.

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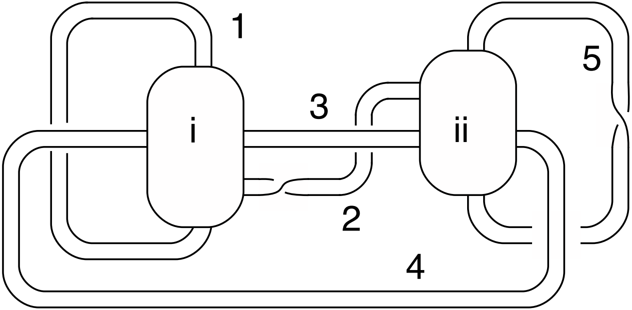

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Riemann mapping theorem. Thurston geometrization: gives pieces with 8 possible geometries glued together along torii, or spheres (Field's Medal 1982)

Signature of Riemann surface: Going up and going down

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Riemann surface signature crowd and map crowd don't see things the same way.

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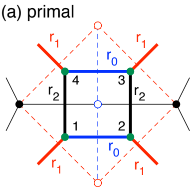

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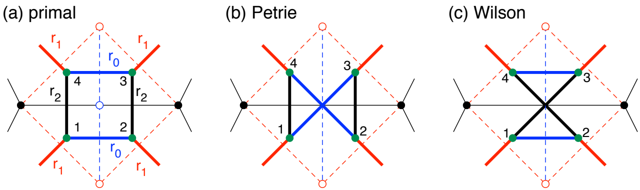

Partial duality: express duality as multiplying r_0 and r_2 by r_0r_2.For partial duality on subset A of edges, only multiply by partial permutation $r_0r_2|A$ (GT 2021 - turn the crank with Grey code for polynomials)

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Proof 3:Go to universal covering U where $M = U/C$ for some subgroup C of universal group. Then go to orientation-preserving subgroup of C.

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