First Workshop on

Constructions of Expanders and Extremal Graphs

List of Abstracts

Slovakia-Austria Bilateral Collaboration

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The following is the list of abstracts for talks presented at the workshop ordered chronologically as presented.

Tuesday, August 27, 14:00

Markus Steenbock, University of Vienna

Rips-Segev torsion-free groups without the unique product property

Abstract: The unique product property is a group property that implies the Kaplansky zero-divisor conjecture, which states that the group algebra of a torsion-free group does not have any non-trivial zero-divisors. I will explain and discuss Rips-Segev's construction of torsion-free groups without the unique product property: the construction is via certain large girth labeled graphs of bounded vertex degree. If the labeled graph admits a compatible wall structure, then the zero divisor conjecture holds for the corresponding group. The existence of such a compatible wall structure on Rips-Segev's graphs is open. In fact, the Rips-Segev construction is currently the only source of potential counter-examples to the zero-divisor conjecture.

Tuesday, August 27, 15:30

Robert Jajcay, Comenius University

Extremal Graphs with Expander Properties

Abstract: The *Degree/Diameter* and the *Cage Problems* are both parts of Extremal Graph Theory aimed at finding extremal k-regular graphs of given diameter or given girth, respectively. Even though the first problem calls for finding *largest* graphs of maximum degree Δ and diameter d and the focus of the second problem is on finding *smallest* k-regular graphs of girth g, the two problems are connected through the so-called *Moore bound* which is simultaneously a lower bound on the order of the smallest k-regular graphs of girth g and an upper bound on the orders of the largest graphs of maximum degree Δ and diameter d.

In our talk, we present another fundamental connection between these two classes of extremal graphs by examining the expander properties of infinite families of near extremal graphs from both classes. We show that in both cases, graphs whose orders are sufficiently close to the Moore bound (the existence of which is at this point only hypothetical) would necessarily form *expander families*; families of graphs of high connectivity. In particular, we show that graphs of maximum degree Δ and diameter d, whose orders only differ from the Moore bound by a constant, would necessarily constitute a family of *Ramanujan graphs*, general constructions of which are relatively rare. Analogously, we show that an infinite family of k-regular graphs of girth g whose orders would not exceed the two multiple of the corresponding Moore bound would form an expanding family.

The results discussed in the Degree/Diameter part of the talk come from joint work with Slobodan Filipovski. The results concerning the Cage Problem are joint work with Leonard Chidiebere Eze.

Tuesday, August 27, 16:30

Shasha Zheng, Comenius University

Cubic graphical regular representations of finite non-abelian simple groups

Abstract: A Cayley graph whose group of symmetries is as small as its underlying group is called a graphical regular representation (GRR for short) of the group. In this talk, we study cubic GRRs of some families of classical groups and, based on some previously known results, confirm that almost every finite non-abelian simple group is the symmetry group of a certain cubic Cayley graph.

Wednesday, August 28, 10:00

Merlin Medici, University of Vienna

Automorphisms of self-similar trees and searching for expanders

Abstract: We give an introduction to self-similar trees. This is a class of trees that naturally appears in a variety of contexts and generalises the notion of regular trees. We then explain how one can describe automorphisms of such trees in explicit terms. Finally, we discuss a construction of an expanding sequence of graphs on regular trees with a view towards applications on self-similar trees.

Wednesday, August 28, 11:30

Pavol Kollár, Comenius University

Regular Families of Permutations - Properties, Counting, and More

Abstract: Cayley graphs form a class of graphs that is well behaved and extensively studied. An example of this "well behavedness" is Sabidussi's result, which characterises Cayley graphs based on properties of their automorphism groups. However, many graphs are not Cayley, even if they are symmetrical enough to be vertex transitive. Because of this, several measures of Cayley-likeness for graphs have been studied, for example in the 2019 paper by Jajcay and Jones *r-regular families of graph automorphisms.*

Our talk is devoted to the related topic of the so-called *r-regular families of permutations of n elements*. We will present the basic definitions, introduce the context in which these families arise, and explain what they tell us about the graph structure, arriving at a similar characterisation theorem to that of Sabidussi. We also further examine properties of these families and investigate methods that help us estimate the number of these structures for fixed parameters r and n. One such estimation is an algorithm producing upper bounds. Another one is based on a probabilistic algorithm achieving a heuristic estimation of the true count.

Wednesday, August 28, 14:00

Tatiana Jajcayová, Comenius University

On the use of partial automorphisms in graph problems

Abstract: A partial automorphism of a graph is an isomorphism between its induced subgraphs. The set of all partial automorphisms of a given graph forms an inverse monoid under composition of partial maps and taking partial inverses. In our presentation, we describe the algebraic structure of such inverse monoids by the means of the standard tools of inverse semigroup theory and give a characterization of inverse monoids which arise as inverse monoids of partial graph automorphisms.

The problem of determining the full automorphism group of a graph is one of the well-known hard problems. The focus of our project is on an extension of the automorphism group problem to that of inverse monoid problem. The full inverse monoid of partial automorphisms of a graph is a much richer algebraical structure that contains more detailed local information. The goal is to apply the algebraic methods of partial permutation semigroup theory to the class of graphs that admit none or only very few total automorphisms and resist the use of methods from permutation group theory. The results involving partial automorphisms and use of inverse monoids may offer new insights into some well-known and long open problems from Graph Theory, as we will illustrate with examples.

Wednesday, August 28, 15:00

Ján Pastorek, Comenius University

Search for correspondences between operations on partial automorphisms and k-dimensional Weisfeiler-Leman algorithm

Abstract: We explore correspondences between operations on partial automorphisms

of a graph and the k-dimensional Weisfeiler-Leman (k-WL) algorithm; a classical combinatorial algorithm for graph isomorphism testing. Partial automorphisms are isomorphisms between induced subgraphs of a given graph, and the set of all partial automorphisms of a graph forms an inverse monoid, the semigroup properties of which were fully characterized by Jajcay et al. One link between the concepts of automorphisms and partial automorphisms can be established by considering extensions of partial automorphisms. We present a bottom-up algorithm for constructing the partial automorphism monoid of a graph and show how this algorithm can be reduced to vertex coloring; thereby establishing a connection to 1-WL. Subsequently, we extend this relation to k-WL. We verified this reduction experimentally for $k < 3$ on graphs of order up to 10. Finally, we discuss potential next steps towards establishing the connection between operations on partial automorphisms of rank k+1 and the k-dimensional Weisfeiler-Leman algorithm k-WL.

Wednesday, August 28, 15:30

Štefánia Glevitzká, Comenius University

Vertex-transitive closures of graphs

Abstract: A graph G is vertex-transitive if its automorphism group acts transitively on its vertices. A vertex-transitive closure of a graph G is a vertex-transitive graph containing G as a spanning subgraph. The vertex-transitive number of G is the smallest possible degree of its vertex-transitive closure. In our talk, we investigate these (relatively new) concepts. We bound and in some cases also determine vertex-transitive numbers for specific classes of graphs, prove some general observations, and investigate a possible relation to automorphism groups.