universität wien

Fakultät für Mathematik

The OeAD Austria-Slovakia cooperation project

« Constructions of expanders and extremal graphs »

WORKSHOP

12.11.2024, SR08, 15:00-16:45 Tatiana Jajcayova (Bratislava)

Combinatorial methods in inverse semigroups

In the first part of our talk, we will review some of the combinatorial methods used in inverse semigroup theory.
Inverse semigroups can be viewed as natural generalizations of groups: While every group can be represented via permutations (one-to-one transformations), inverse semigroups can be represented via partial one-to-one transformations. We will introduce graphs (automata), called Schützenberger automata, that are related to presentations of inverse semigroups in a way similar to the way Cayley graphs are related to groups. As shown by the combinatorial approach introduced by Munn and extended by Stephen, these automata are instrumental in the study of structural and algorithmic questions concerning inverse semigroup presentations.
In the second part of the talk, we will turn our attention to HNN extensions of inverse semigroups. HNN extensions, a clasical construction originally introduced in group theory, also proved useful in the study of decidability questions in the class of inverse semigroups. We study these extensions via the structure of their Schützenberger automata. These automata are in general infinite, but in specific cases we can detect so called finite core - in the sense that all important information about the automaton is encoded in a “finite” subgraph. This nice property yields in some cases an effective construction of the Schützenberger automata and thus allows us to answer algorithmic questions (such as the Word Problem) about presentations of certain classes of inverse semigroups.
If time permits, we will also consider actions of special groups on Schützenberger graphs, which will allow us to employ the powerful Bass-Serre theory, and to consider structural questions concerning HNN extensions.

13.11.2024, SR06, 09:15-10:00 Cristopher Cashen (Vienna)

Biggs colored tree groups containing the alternating group

Biggs gave a construction of families of Cayley graph of permutation groups with fixed degree and unbounded girth in terms of colored trees. We give sufficient conditions, in terms of the radius of the tree and number of colors, for the group so defined to contain the alternating group. The conditions cover infinitely many radii for each number of colors greater than 2, and cover all ‘small’ cases. This gives theoretical justification for computer experimental results of Exoo and Jajcay.

13.11.2024, SR06, 10:15-11:00 Dominika Závacká (Bratislava)

Algorithmic approach to obtaining values of (k,g)-spectra

A (k,g)-graph is a k-regular graph of girth g. The (k,g)-spectrum is the set of all possible orders of (k,g)-graphs for a specific degree/girth pair (k,g). The smallest order in the (k,g)-spectrum corresponds to the order n(k,g) of a (k,g)-cage, which is a graph of the smallest order among all (k,g)-graphs. The value n(k,g) is unknown and difficult to determine for most parameter pairs (k,g), and the general problem of determining these values for all parameter pairs (k,g) is the well-known Cage Problem. In our talk, we will present various methods for identifying orders contained in the (k,g)-spectra; which is equivalent to proving the existence or constructing (k,g)-graphs of prescribed orders. It is interesting to note that a (k,g)-spectrum is not always continuous; for instance, the (3,8)-spectrum contains values 30 and 34, but lacks the order 32; as it has been computationally proven that no (3,8)-graph of order 32 exists. To address this, we define and compute N(k,g), the smallest order from which a (k,g)-graph of every larger order is guaranteed to exist. As determining the smallest value n(k,g) of a (k, g)-spectrum is a computationally hard problem, we focus on cases where the minimal order n(k,g) (the order of a (k,g)-cage) is already known. Despite the challenges of establishing the full (k,g)-spectra for general pairs (k,g), we provide both complete and partial computation results for several parameter pairs (k,g).

13.11.2024, BZ09, 14:00-14:45 Goulnara Arzhantseva (Vienna)

Large girth graphs with bounded diameter-by-girth ratio

We discuss explicit constructions of graphs as in the title. We explain current group-theoretical applications and formulate some open questions.