Graphs that are Cayley on more than one group

Joy Morris

University of Lethbridge

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Algebraic Graph Theory International Webinar

Ontheworldmap.com
Arctic Ocean
Greenland
(Denmark)
Baffin Bay
Resolute
Alaska
(United States)
Inuvik
Cambridge Bay
Dawson
Yukon
Whitehorse
Echo Bay
Northwest
Territories
Nunavut
Iqaluit
Labrador Sea
Watson Lake
Yellowknife
Rankin Inlet
Ivujivik
Hay River
Fort Nelson
Fort Smith
Hudson
Bay
Newfoundland
and
Labrador
Happy Valley
Schefferville
Gander
St. John's
British
Columbia
Prince Rupert
Churchill
Saskatchewan
Prince George
Grande Prairie
Alberta
Edmonton
Manitoba
Flin Flon
Chisasibi
Sept-Îles
Quebec
Gulf of Saint
Lawrence
Saint Pierre
and Miquelon
(France)
Prince Edward
Island
Sydney
Charlottetown
Pacific Ocean
Vancouver
Victoria
Kelowna
Red Deer
Calgary
Lethbridge
Prince Albert
Saskatoon
Regina
Brandon
Winnipeg
Moosonee
Chibougamau
Ontario
New
Brunswick
Fredericton
Moncton
Halifax
Saint John
Nova
Scotia
Montreal
Sherbrooke
Thunder Bay
Lake Superior
Sudbury
Ottawa
Lake Huron
Lake Ontario
Toronto
Hamilton
London
Lake Michigan
Lake Erie
Atlantic
Ocean
United States

Overview

Introduction

Disclaimer

Throughout this talk, assume groups and graphs are finite.

Circulant graphs

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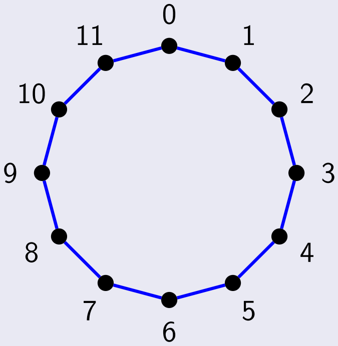

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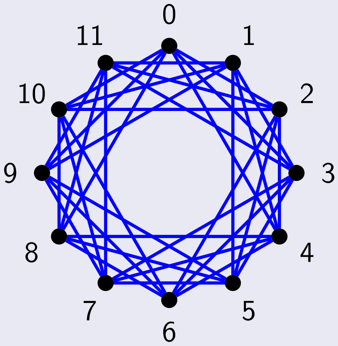

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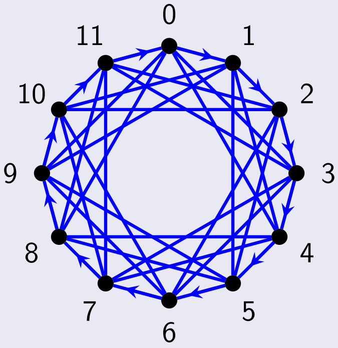

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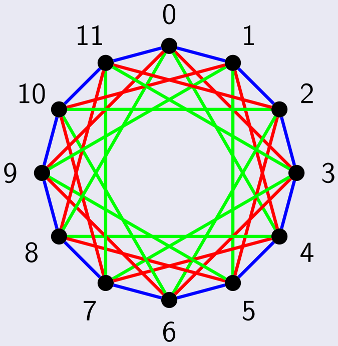

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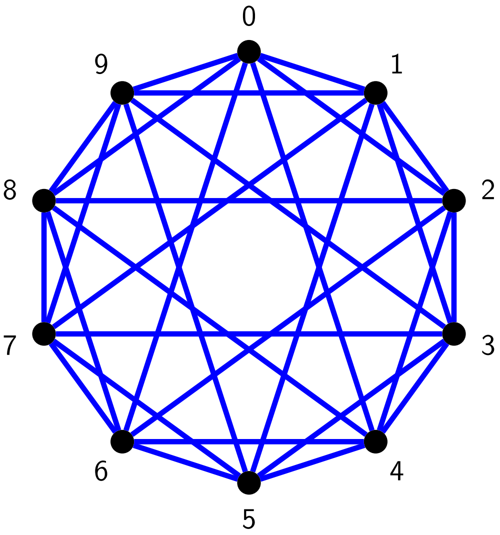

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Cayley graphs

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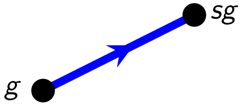

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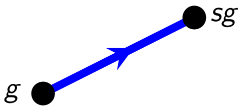

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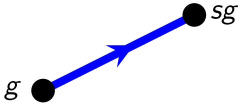
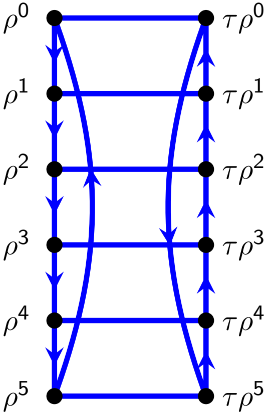


Notice

- Γ will be a graph if and only if $S = S^{-1}$;
- right-multiplication by any element of G is necessarily an automorphism of this (di)graph (there is an arc from gh to sgh).

Example: Cay($D_6, \{\rho, \tau\}$)



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Cayley graphs and Regular groups of Automorphisms

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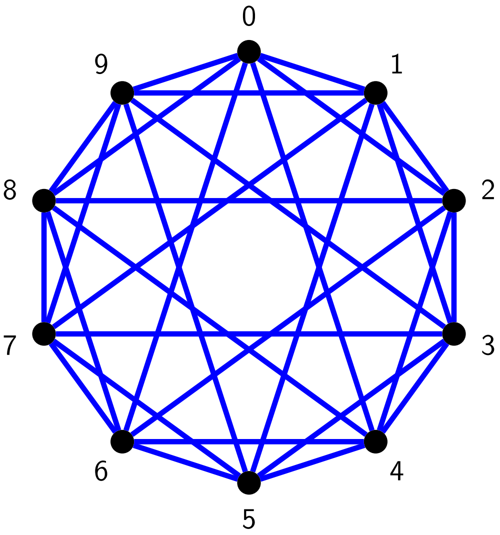
So if a graph has more than one regular subgroup in its automorphism group, it can be represented in more than one way as a Cayley graph.

Cayley graphs on different groups - warm-up examples



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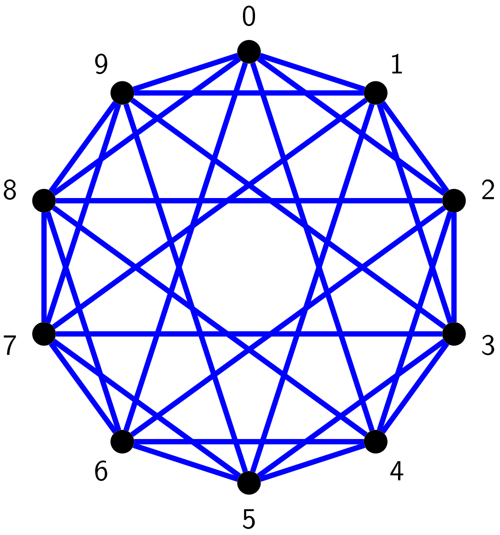

$n = 10$, $S = \pm\{1, 2, 4\}$. $D_{10} \leq \text{Aut}(\Gamma)$, including a regular action of D_5.

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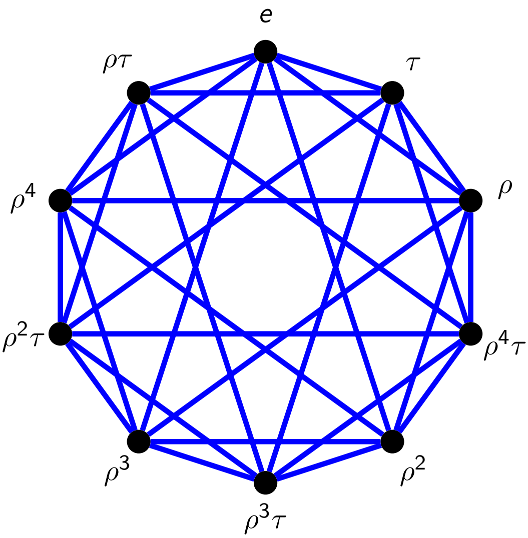

$D_5,\ S = \{\rho^{\pm 1}, \rho^{\pm 2}, \tau, \rho\tau\}$

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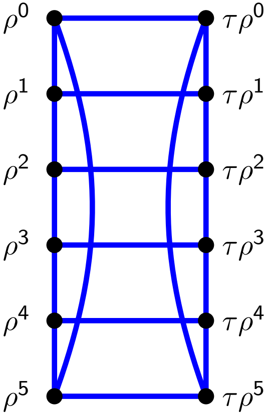

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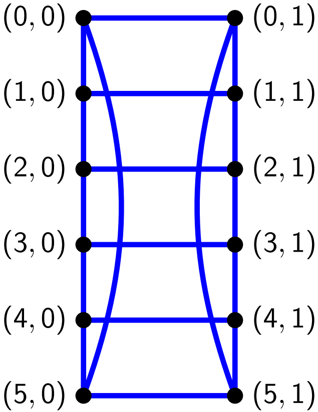

$\mathbb{Z}_6 \times \mathbb{Z}_2$, $S = \{(1,0), (5,0), (0,1)\}$

Understanding these examples

Proposition (M., Smolčić, 2020)

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A Cayley graph on $\mathrm{Dih}(A, x)$ will also be a Cayley graph on $A \times \mathbb{Z}_2$ if there exists some $y \in xA$ such that for every $a \in A$ we have $ya \in S \cap xA$ if and only if $ya^{-1} \in S \cap xA$.

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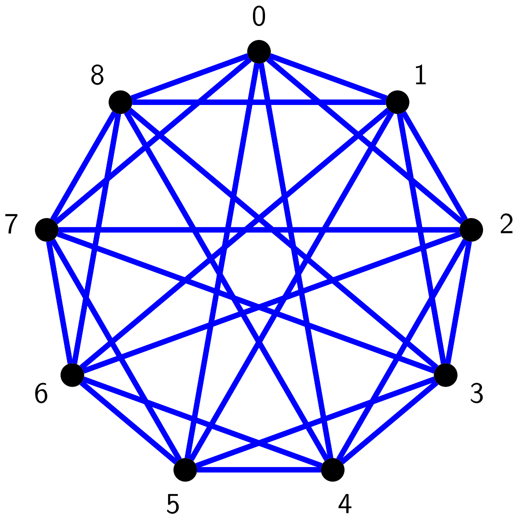
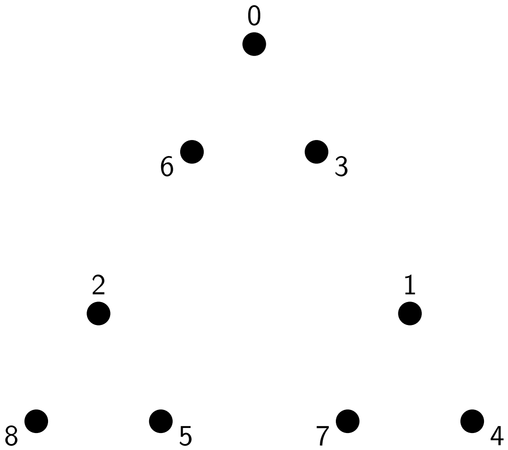
Circulants of prime power order and Lexicographic Products

Example



$n = 9$, $S = \{1, 2, 4, 5, 7, 8\}$.

Example redrawn



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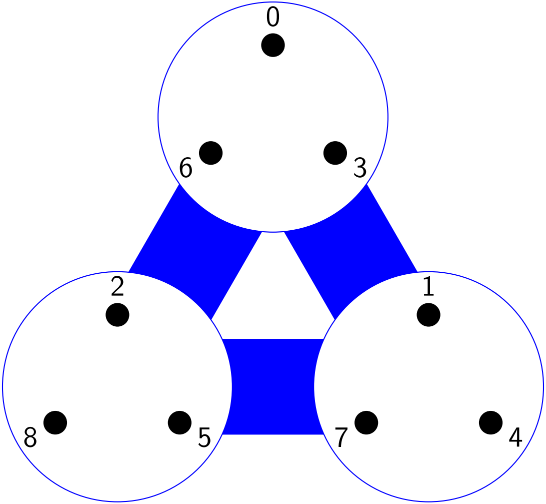

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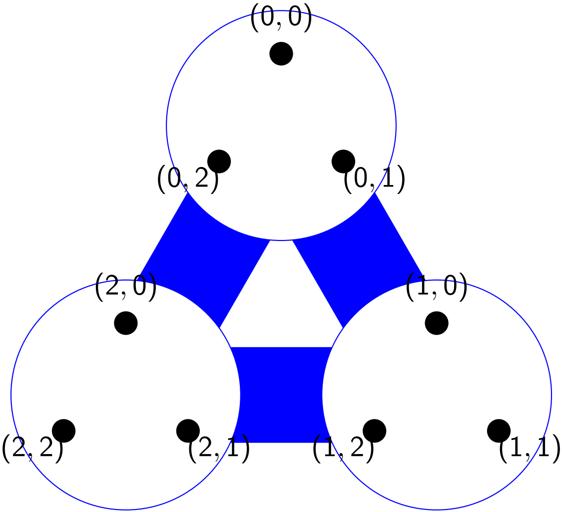

$\mathbb{Z}_3 \times \mathbb{Z}_3,\ S = \{(1,0),(1,1),(1,2),(2,0),(2,1),(2,2)\}.$

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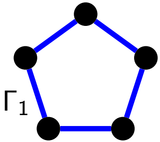

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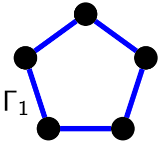

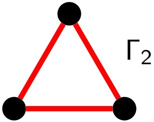

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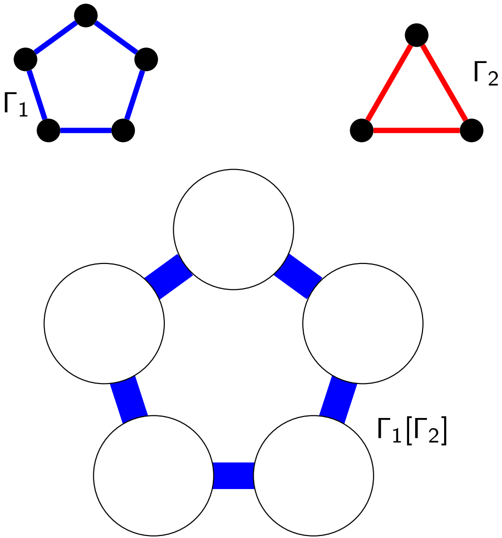

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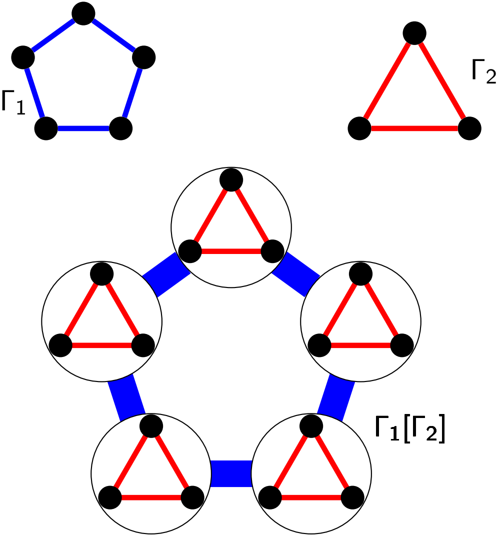

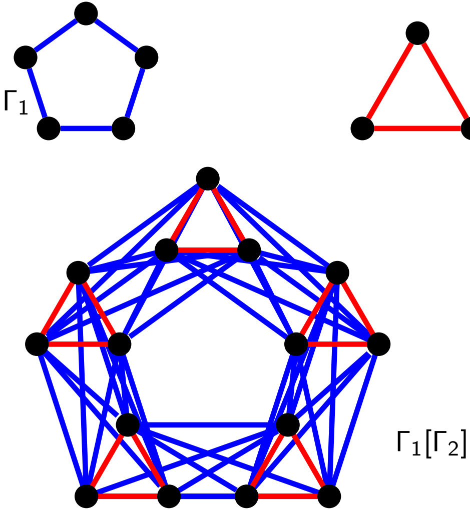
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$\Gamma_1[\Gamma_2]$

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- $Aut(\Gamma) \cong Aut(\Gamma_1') \wr Aut(\Gamma_2')$.

In general, $\Gamma_1' = \Gamma_1$.

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Proposition

A Cayley graph $\mathrm{Cay}(G, S)$ is a lexicographic product if and only if S is a union of right cosets of some $H \leq G$.

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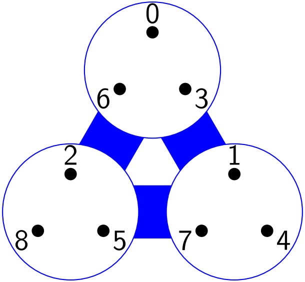
Recall our example:



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Anne Joseph O'Connell

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The graph $\Gamma = Circ(2^n, S)$ is also a Cayley graph of some other abelian group of order 2^n if and only if Γ is a lexicographic product. Abelian is necessary, since $Circ(2^n, S)$ is always a Cayley graph on $D_{2^{n-1}}$.

Istvan Kovács

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Whereas the automorphism of a circulant digraph on p^n vertices could be as small as p^n.

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In fact, typically these graphs will be Cayley on lots of groups.

Other results

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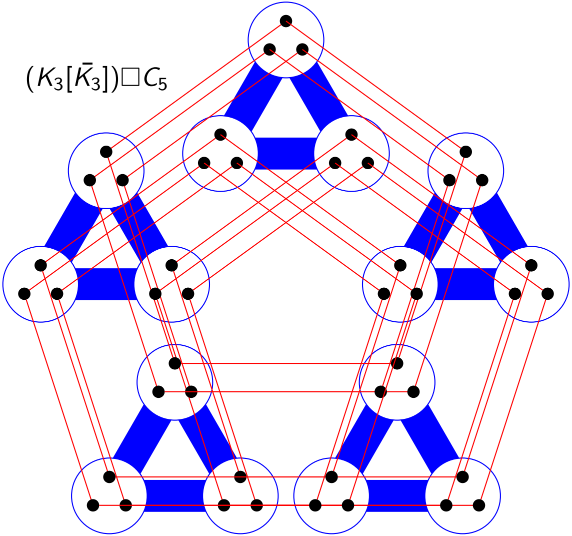
Conjecture

The condition $\gcd(k, \varphi(k)) = 1$ is not necessary.

Example

$\text{Circ}(45, \pm\{5, 10, 20, 9\}) \cong \text{Cay}(\mathbb{Z}_3 \times \mathbb{Z}_{15}, \pm\{(1,0),(1,5),(1,10),(0,6)\})$.



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Thank you!

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