Symmetric substructures in tetravalent edge-transitive bicirculant graphs

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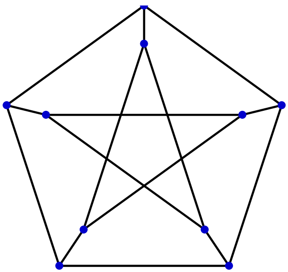
G-transitive graphs

The graph Γ is said to be ***G*-vertex-**, ***G*-edge-** and ***G*-arc-transitive** for some $G \leq \mathrm{Aut}(\Gamma)$ if G acts transitively on $V(\Gamma)$, $E(\Gamma)$ and $A(\Gamma)$, respectively.

In the case of $G = \mathrm{Aut}(\Gamma)$, we omit the prefix G and simply write vertex-transitive, edge-transitive and arc-transitive.

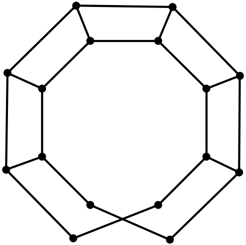
Cubic vertex-transitive graphs

In the area of symmetries of graphs, **finite connected 3-regular vertex-transitive graphs** (cubic vertex-transitive graphs, CVT) play a very special role.



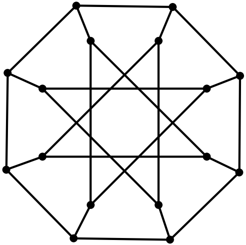
- Trivalent symmetric graphs on up to 768 vertices,
- Cubic vertex-transitive graphs on up to 1280 vertices,
- Semiregular automorphisms of vertex-transitive cubic graphs,
- Hamiltonian cycles in cubic Cayley graphs,
- Cubic arc-transitive k-multicirculants,
- Bounding the order of the vertex-stabiliser in 3-valent vertex-transitive...,
- Symmetry properties of generalized graph truncations,
- Non-Cayley vertex-transitive graphs of order twice the product of two odd primes,
- ...

Cubic vertex-transitive graphs - Families



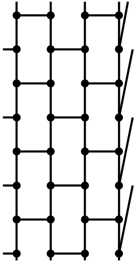
Möbius ladder

M_n



Generalized Petersen

$GP(n, k)$



Honeycomb toroidal

$HTG(m, n, l)$

Cubic vertex-transitive graphs - Families

1. Prisms
2. Double generalised Petersen graphs
3. Split Praegex-Xu graphs
4. Honeycomb toroidal.
5. Cyclic Haar graphs.
6. Möbius ladder
7. Tricirculants of Type ...
8. ...

Cubic vertex-transitive graphs - Properties

1. for which parameters the property X holds?
2. Girth?
3. Is it a Cayley graph?
4. Edge-signature?
5. Maps?
6. Automorphism group

Cubic vertex-transitive graphs - DATABASE

$CVTinfo[34,1] := [6, Haar(17,1,4)]$
$CVTinfo[34,2] := [4, Moeb(17), Haar(17,1,2), MapT3b_((8,1)]$
$CVTinfo[34,3] := [6, Haar(17,1,3), MapP_1 7]$
$CVTinfo[34,4] := [4, Prism(17), GenPet(17,1)]$
$CVTinfo[34,5] := [7, GenPet(17,4)]$
$CVTinfo[36,1] := [4, Moeb(18)]$
$CVTinfo[36,2] := [7]$
$CVTinfo[36,3] := [6, Haar(18,1,3), MapP_1 8]$
$CVTinfo[36,4] := [4, Haar(18,1,9), Gamma_((9)]$
$CVTinfo[36,5] := [6, Haar(18,1,6), MapD_6]$
$CVTinfo[36,6] := [6, Haar(18,1,5)]$
$CVTinfo[36,7] := [6, Haar(18,1,4)]$
$CVTinfo[36,8] := [4, Prism(18), GenPet(18,1), Haar(18,1,2), MapT3a_((9,1)]$
$CVTinfo[36,9] := [4]$
$CVTinfo[36,10] := [6, MapS_6, MapT3a_((3,3)]$
$CVTinfo[36,11] := [6]$
$CVTinfo[36,12] := [4]$
$CVTinfo[38,1] := [4, Moeb(19), Haar(19,1,2), MapT3b_((9,1)]$
$CVTinfo[38,2] := [6, Haar(19,1,8), MapT2_((3,2)]$
$CVTinfo[38,3] := [6, Haar(19,1,3), MapP_1 9]$
$CVTinfo[38,4] := [6, Haar(19,1,4)]$
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$CVTinfo[40,1] := [4, Moeb(20)]$
$CVTinfo[40,2] := [4, Haar(20,1,10), MapT3a_((2,5), Gamma_((10)]$
$CVTinfo[40,3] := [6, GenPet(20,9), MapT3b_((4,2)]$
$CVTinfo[40,4] := [6, Haar(20,1,4)]$

Prisms

- **Name:** `Prism`
- **Parameters:** n; $n \in \mathbb{Z}$, $n \geq 3$.
- **Vertex-set:** $\mathbb{Z}_n \times \mathbb{Z}_2$.
- **Edge-set:** $E = E_1 \cup E_2$,
  $E_1 = \{\{(x,0),(x,1)\} : x \in \mathbb{Z}_n\}$,
  $E_2 = \{\{(x,i),(x+1,i)\} : x \in \mathbb{Z}_n\}$.
- **Vertex-transitivity:** All Prisms are know to be vertex-transitive.
- **Comments:** $\mathrm{Aut}(\mathrm{Prism}(n))$ is isomorphic either to $\mathrm{D}_n \times \mathrm{C}_2$ if $n \neq 4$ or to $\mathrm{Sym}(4) \times \mathrm{C}_2$ if $n = 4$; note that in the latter case, the prism is in fact isomorphic to the skeleton of the 3-dimensional cube.

Möbius ladder

- **Name:** `Moeb`
- **Parameters:** n; $n \in \mathbb{Z}$, $n \geq 2$.
- **Vertex-set:** $\mathbb{Z}_{2n}$.
- **Edge-set:** $E = E_1 \cup E_2$,
  $E_1 = \{\{x, x+1\} : x \in \mathbb{Z}_{2n}\}$,
  $E_2 = \{\{x, x+n)\} : x \in \mathbb{Z}_{2n}\}$.
- **Vertex-transitivity:** All Möbius ladders are known to be vertex-transitive.
- **Comments:** Unless $n \in \{2,3\}$, the automorphism group of $\mathrm{Moeb}(n)$ is isomorphic to the dihedral group D_{2n} of order $4n$, having two orbits on the edges of $\mathrm{Moeb}(n)$. On the other hand, $\mathrm{Moeb}(2) \cong \mathrm{K}_4$ and $\mathrm{Moeb}(3) \cong \mathrm{K}_{3,3}$, a complete bipartite graph, are both arc-transitive.

Generalised Petersen graphs

- **Name:** `GP`
- **Parameters:** n, k; $n, k \in \mathbb{Z}$, $n \geq 3$, $1 \leq k \leq \frac{n}{2}$.
- **Vertex-set:** $\mathbb{Z}_n \times \mathbb{Z}_2$.
- **Edge-set:** $E = E_1 \cup E_2 \cup E_3$,

How can we get families of CVT with certain properties?

Cubic vertex-transitive graphs

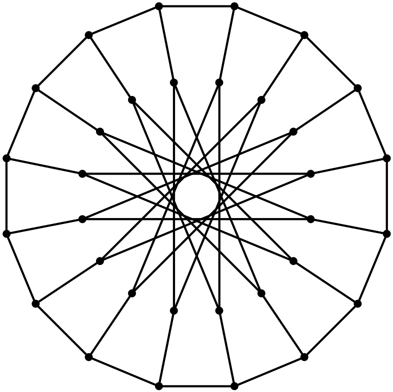
Let Γ be a cubic graph with $G \leq \mathrm{Aut}(\Gamma)$ acting transitively on $\mathrm{V}(\Gamma)$. Fix a vertex $v \in \mathrm{V}(\Gamma)$ and consider the permutation group $G_v^{\Gamma(v)}$ induced by the action of the stabiliser G_v on the neighbourhood $\Gamma(v)$.

1. If $G_v^{\Gamma(v)}$ is transitive, then G acts transitively on the arc-set $\mathrm{A}(\Gamma)$.
2. If $G_v^{\Gamma(v)}$ is a trivial group, then the assumed connectivity of Γ implies that G_v is trivial and hence that G acts regularly on $\mathrm{V}(\Gamma)$. If we have taken G to be equal to $\mathrm{Aut}(\Gamma)$, then Γ is in fact a *graphical regular representation* of G, or a *zero-symmetric graph*.
3. G is of **Type** 2^* (2 orbits on the set of arcs).

Cubic vertex-transitive graphs - Type 2*

- Let Γ be a cubic vertex-transitive graph admitting a group $G \leq \mathrm{Aut}(\Gamma)$ with exactly two arc-orbits.
- Let $v \in \mathrm{V}(\Gamma)$ and note that the stabiliser G_v has two orbits on the arcs incident to v. That is, a non-trivial element of G_v interchanges two of the arcs incident to v, while fixing the third one.
- Let x be the arc incident to v that is fixed by G_v, and let $M = x^G$. The set M is a matching and the two orbits of G on the arcs of Γ are precisely M and $\mathrm{A}(\Gamma) \setminus M$.
- If Γ is neither a prism or a Möbius ladder, then the merge $\Gamma[M]$ must be a simple graph.

Cubic vertex-transitive graphs - Type 2^* - $GP(16, 7)$



Cubic vertex-transitive graphs - Type 2^* - $GP(16,7)$

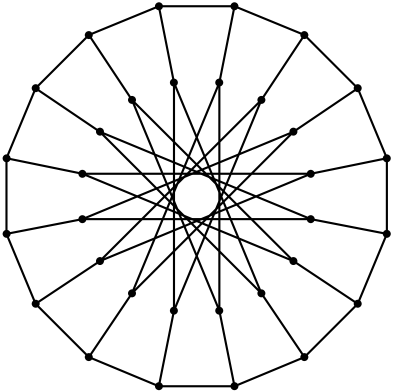
Theorem

Let $\Gamma = \mathrm{GP}(n,k)$ *be a generalized Petersen graph. Then*

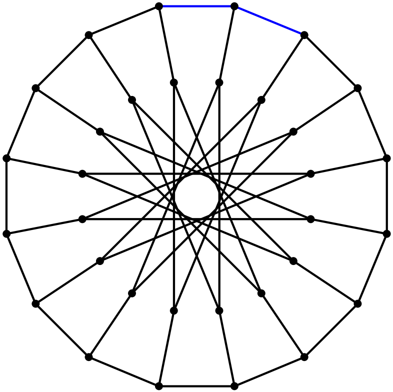
1. *it is symmetric if and only if* $(n,k) \in \{(4,1),(5,2),(8,3),(10,2),(10,3),(12,5),(24,5)\}$,
2. *it is vertex-transitive if and only if* $k^2 \equiv \pm 1 \pmod{n}$ *or if* $n = 10$ *and* $k = 2$,
3. *it is a Cayley graph if and only if* $k^2 \equiv 1 \pmod{n}$.

- Frucht, R., Graver, J. E., Watkins, M. E., *The groups of the generalized Petersen graphs*, Math. Proc. Camb. Philos. Soc., **Vol. 70**, Cambridge Univ. Press, (1971) 211–218.
- Nedela, R. , Škoviera, M. [1995], *Which generalized Petersen graphs are Cayley graphs?*, J. Graph Theory **19(1)** (1995), 1–11.
- Saražin, M. L., *A Note on the Generalized Petersen Graphs That Are Also Cayley Graphs*, J. Comb. Theory, Ser. B **69** (1997), 226–229.

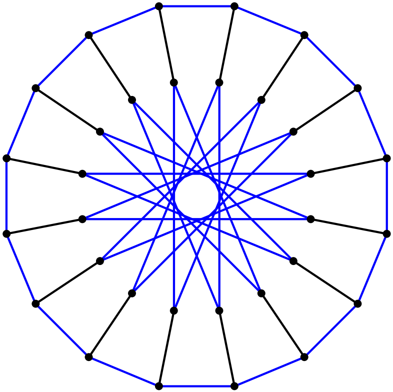
Cubic vertex-transitive graphs - Type 2^* - $GP(16, 7)$



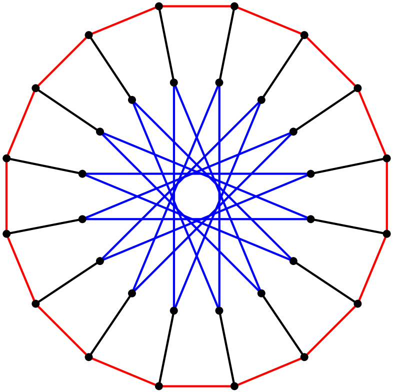
Cubic vertex-transitive graphs - Type 2^* - $GP(16,7)$



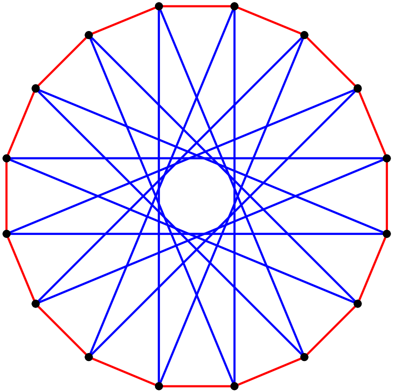
Cubic vertex-transitive graphs - Type 2^* - $GP(16, 7)$



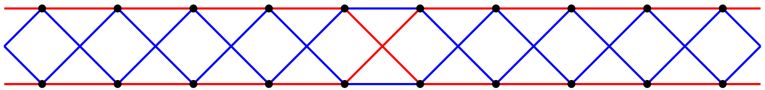
Cubic vertex-transitive graphs - Type 2^* - $GP(16, 7)$



Cubic vertex-transitive graphs - Type 2^* - $GP(16,7) - >$



Cubic vertex-transitive graphs - Type 2^* - $GP(18, 7) ->$



A **cycle structure** in a tetravalent graph Γ is a partition $\mathcal{Y}$ of its edges into cycles such that the subgroup $\mathrm{Aut}(\mathcal{Y})$ of $\mathrm{Aut}(\Gamma)$ which preserves $\mathcal{Y}$ is transitive on the arcs of Γ.

Cubic vertex-transitive graphs - Merge

Let Γ be a k-valent graph with a matching M. Let $\overline{M}$ be the the set of edges containing an arc in M. The **merge** of Γ by M is the graph $\Gamma[M](\Gamma) = (\mathrm{V}', \mathrm{D}', \mathrm{beg}', \mathrm{inv}')$ such that:

1. $\mathrm{V}' = \overline{M}$;
2. $\mathrm{D}' = \mathrm{D}(\Gamma) \setminus M$;
3. $\mathrm{beg}'\, x$ is the unique edge $\{y, y^{-1}\} \in \overline{M}$ with $\mathrm{beg}_\Gamma\, x \in \{\mathrm{beg}_\Gamma\, y, \mathrm{beg}_\Gamma\, y^{-1}\}$;
4. $\mathrm{inv}'\, x = \mathrm{inv}_\Gamma\, x$.

Informally, Λ is the graph obtained by merging the endvertices (contracting) of every edge in $\overline{M}$.

CVT

CVT Type 2*

$\Downarrow$

Arc-transitive 4-valent graph
CS

Constructing cubic vertex-transitive graphs

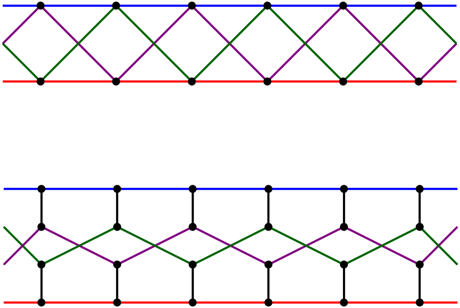
CVT Type 2*

$\Uparrow$

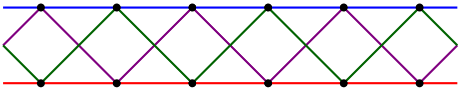
Arc-transitive 4-valent graph
CS

Constructing cubic vertex-transitive graphs - $\mathrm{PX}(6,1)$

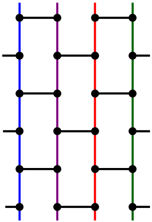
Cycle structure in a tetravalent graph Γ:



Constructing cubic vertex-transitive graphs



HTG(4, 6, 0)



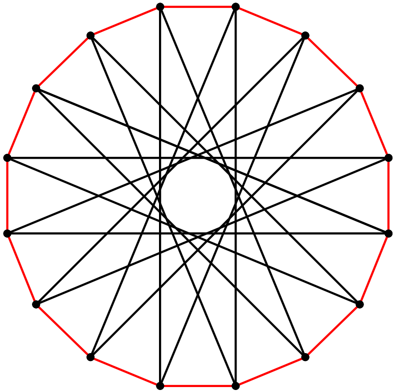
Splits of cycle structure

The input of this construction is a pair $(\Gamma, \mathcal{Y})$, where Γ is a tetravalent arc-transitive graph and $\mathcal{Y}$ is an arc-transitive cycle decomposition of Γ. The output is the graph $\mathrm{Sp}(\Gamma, \mathcal{Y})$, the vertices of which are the pairs (v, C) where $v \in V(\Gamma)$, $C \in \mathcal{Y}$ and v lies on the cycle C, and two vertices (v_1, C_1) and (v_2, C_2) are adjacent if and only if either $v_1 = v_2$ and $C_1 \neq C_2$, or $C_1 = C_2$ and v_1, v_2 is an edge of C_1. Note that the set of edges of the form $\{(v, C_1), (v, C_2)\}$ constitute a perfect matching, $M_{\mathcal{Y}}$.

Constructing cubic vertex-transitigve graphs

Consistent cycles

Let Γ be a graph admitting an arc-transitive group of automorphisms $G \leq \mathrm{Aut}(\Gamma)$.
A directed (but not rooted) cycle $\vec{C} = (v_0, v_1, \ldots, v_{r-1})$ of Γ is said to be G-**consistent** if there exists $g \in G$ mapping each v_i to v_{i+1} (where the indices are computed modulo r).
In this case g is said to be a **shunt** of $\vec{C}$. Of course, the *inverse* $\vec{C}^{-1} = (v_0, v_{r-1}, v_{r-2}, \ldots, v_1)$ is G-consistent if and only if $\vec{C}$ is G-consistent. **Thus an (undirected) cycle is said to be G-consistent if both of its two corresponding directed cycles are G-consistent.**



Cycle structure

A **cycle structure** in a tetravalent graph Γ is a partition $\mathcal{Y}$ of its edges into cycles such that the subgroup $\mathrm{Aut}(\mathcal{Y})$ of $\mathrm{Aut}(\Gamma)$ which preserves $\mathcal{Y}$ is transitive on the arcs of Γ.

The cycles of a cycle structure $\mathcal{Y}$ must be **consistent** and all of the same length; in fact, they must all be within the same orbit of consistent cycles under $\mathrm{Aut}(\Gamma)$.

Two cycle structures $\mathcal{Y}$ and $\mathcal{Y}'$ in a graph Γ are said to be *isomorphic* if there exists a symmetry of Γ mapping the cycles in $\mathcal{Y}$ to the cycles in $\mathcal{Y}'$.

We will call a cycle structure **bipartite** provided that we can partition the cycles of $\mathcal{Y}$ into two colors, so that each vertex is incident to one cycle of each color.

Cycle structure

Lemma

Let Γ is a tetravalent arc-transitive graph and $\mathcal{Y}$ is an arc-transitive cycle decomposition of Γ. Then $|\mathrm{Aut}(\mathrm{Sp}(\Gamma,\mathcal{Y}))| \geq |\mathrm{Aut}(\mathcal{Y})|$ with the equality holding if and only if $\mathrm{Sp}(\Gamma,\mathcal{Y})$ is not arc-transitive.

$\mathrm{Sp}(\Gamma,\mathcal{Y}))$ is not arc-transitive $\iff$ $|\mathrm{Aut}(\mathrm{Sp}(\Gamma,\mathcal{Y}))| = |\mathrm{Aut}(\mathcal{Y})|$.

Cycle structure

Theorem (Š.M,P.P, S.W.)

Let G be an arc-transitive group of automorphisms of a connected tetravalent graph Γ. Let $\mathrm{cs}(G)$ denote the number of G-invariant cycle decompositions of Γ. Then the following holds:

(i) *If G is 2-arc-transitive (equivalently, $G_v^{\Gamma(v)} \cong A_4$ or S_4), then $\mathrm{cs}(G) = 0$.*

(ii) *If $G_v^{\Gamma(v)} \cong D_4$ or C_4, then $\mathrm{cs}(G) = 1$;*

(iii) *If $G_v^{\Gamma(v)} \cong C_2 \times C_2$, then $\mathrm{cs}(G) = 3$.*

Constructing cubic vertex-transitive graphs

1. (Family) tetravalent arc-transitive graph(s).
2. Orbits of consistent cycles.
3. Cycle structures.
4. The split graphs.
5. (Family) Cubic vertex transitive graph.

Tetravalent arc-transitive graphs

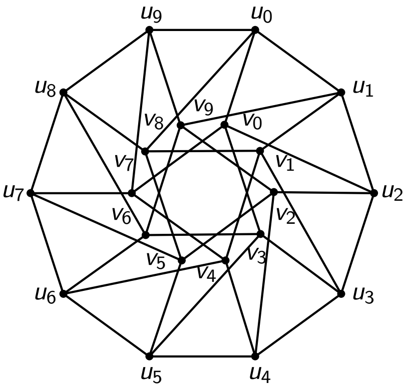
Tetravalent Bicirculant graphs

A graph Γ is said to be **bicirculant** provided that it has a symmetry ρ which moves its $2n$ vertices in two cycles, each of length n.

We let u_i, v_i for $i \in \mathbb{Z}_n$ be its vertices and assume that

$$\rho = (u_0, u_1, u_2 \ldots u_{-2}, u_{-1})$$

$$(v_0, v_1, v_2 \ldots v_{-2}, v_{-1}).$$



Tetravalent Edge-Transitive Bicirculant graphs

There are two families of such graphs:
(I. Kovács, B. Kuzman, A. Malnič, S. Wilson)

1. **The Rose Window graphs**. Here, the graph $\mathrm{R}_n(a, r)$ has four kinds of edges
   - (a $\{u_i, u_{i+1}\}$
   - (b $\{u_i, v_i\}$
   - (c $\{v_i, u_{i+a}\}$
   - (d $\{v_i, v_{i+r}\}$
2. **The bipartite dihedrants**. Here, the graph $\mathrm{BD}_n(0, a, b, c)$ has all edges of the form $\{u_i, v_{i+s}\}$ for $s \in \{0, a, b, c\}$)

The Rose Window graphs - $\mathrm{R}_n(a, r)$

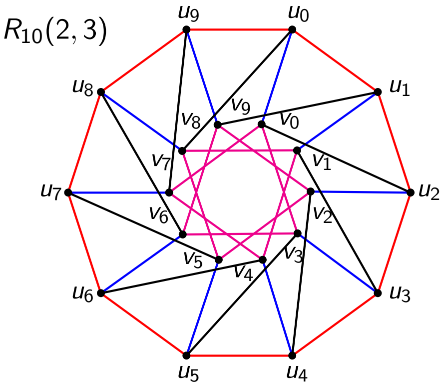
Let n be an integer, $n \geq 3$, and let $a, r \in \mathbb{Z}_n \setminus \{0\}$. The **Rose window graph** $\mathrm{R}_n(a, r)$ is then defined to have the vertex-set $\{u_i : i \in \mathbb{Z}_n\} \cup \{v_i : i \in \mathbb{Z}_n\}$ of cardinality $2n$ and the edges being of four types:

(a) *rim edges*: $\{u_i, u_{i+1}\}$, $i \in \mathbb{Z}_n$;

(b) *in-spokes*: $\{u_i, v_i\}$, $i \in \mathbb{Z}_n$;

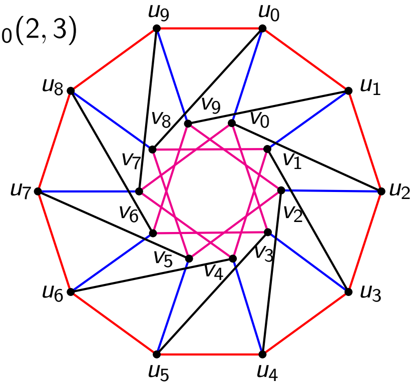
(c) *out-spokes*: $\{v_i, u_{i+a}\}$, $i \in \mathbb{Z}_n$;

(d) *hub edges*: $\{v_i, v_{i+r}\}$, $i \in \mathbb{Z}_n$.



Tetravalent Edge-Transitive Bicirculant graphs - RW

$R_{10}(2,3)$



There are two obvious automorphisms of a graph $\mathrm{R}_n(a,r)$, the rotation

$$\rho = (u_0, u_1, u_2, \ldots, u_{n-2}, u_{n-1})$$

$$(v_0, v_1, v_2 \ldots v_{n-2}, v_{n-1})$$

and the *reflection* μ, which interchanges each u_i with u_{n-i} and each v_i with v_{n-i-a}.

The Rose Window graphs

Theorem (I. Kovács, K. Kutnar, D. Marušič)

An edge-transitive Rose window graph $\mathrm{R}_n(a, r)$ *with* $n \geq 3$, $1 \leq a, r \leq n-1$, *is isomorphic to a member of the following four families:*

(I) $\mathrm{R}_n(2, 1)$ *(*$\mathrm{PX}(n, 1)$*);*

(II) $\mathrm{R}_{2m}(m+2, m+1)$ *(*$\mathrm{PX}(m, 2)$*);*

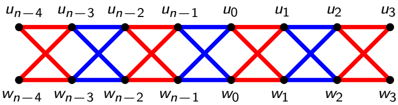
(III) $\mathrm{R}_n(3d+2, 9d+1)$ *where* $n = 12m$ *and* $d = m$ *or* $-m$*;*

(IV) $\mathrm{R}_{2m}(2b, r)$ *where,* $m \geq 3$, $1 \leq b \leq m-1$, $b^2 \equiv \pm 1 \pmod m$ *and either*

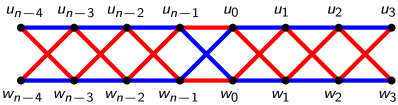
- (i) $r = 1$ *and* $b \notin \{1, m-1\}$, *or*
- (ii) $r = m+1$, m *is even and* $(m \bmod 4, b) \neq (0, \frac{m}{2}+1)$.

Cycle structures

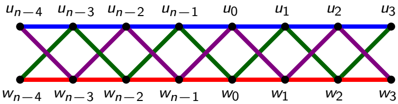
Family (I) - $\mathrm{R}_n(2,1)$ $(\mathrm{PX}(n,1))$:
(R. Jajcay, P. Potočnik, S. Wilson,)



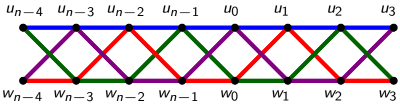
(*i*) $\mathcal{Y}^*$



(*ii*) $\mathcal{Y}'_{10*}$



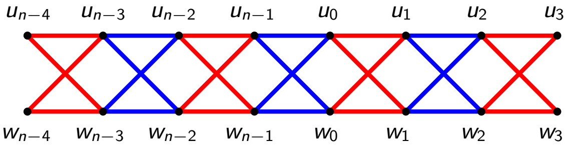
(*iii*) $\mathcal{Y}_{10*}$

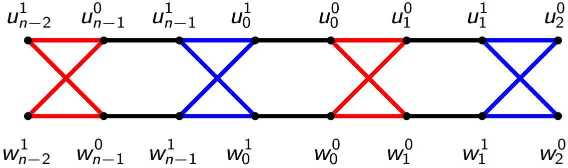


(*iv*) $\mathcal{Y}_{110*}$

Splits of cycle structure

Family (I) - $\mathrm{R}_n(2,1) -> Sp\mathrm{PX}(n,1)$;





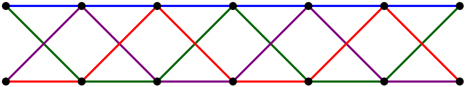
Splits of cycle structure

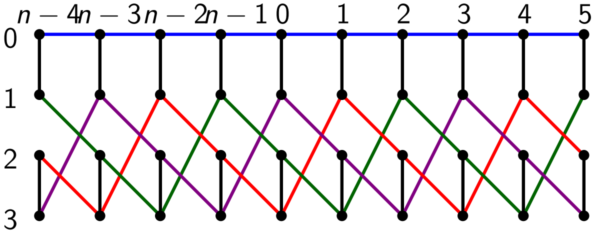
Let $\Gamma = \mathrm{R}_n(2,1)$ and $\Gamma' = \mathrm{Sp}(\Gamma, \mathcal{Y})$. Let $G' = \mathrm{Aut}(\Gamma')$. Here 'CL' stands for the length of the cycles in the cycle structure:

$\mathcal{Y}$	CL	Cond	Bipartite	$\lvert \mathrm{Aut}(\mathcal{Y}) \rvert$	$\Gamma' = \mathrm{Sp}(\Gamma, \mathcal{Y})$	$Girth(\Gamma')$	ESignature	Bip
$\mathcal{Y}^*$	4	–	if n even	$2n(2^n)$	$\Gamma(n)$ $\mathrm{HTG}(1, 4n, 2n-1)$	4	$(0, 1, 1)$	Yes
$\mathcal{Y}'_{10*}$	$2n$	$2 \mid n$	Yes	$8n$	$\mathrm{GP}(2n, n-1)$ $\mathrm{HTG}(2, 2n, n)$	6	$(2, 2, 2)$	Yes
$\mathcal{Y}_{10*}$	n	$2 \mid n$	Yes	$8n$	$\mathrm{HTG}(4,\mathrm{n},0)$	6	$(2, 2, 2)$	Yes
$\mathcal{Y}_{110*}$	n	$3 \mid n$	No	$8n$		7	$(4, 4, 6)$	No

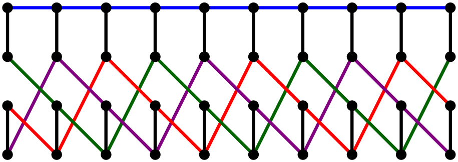
Table: Cycle Structures in $\mathrm{R}_n(2,1) = \mathrm{PX}(n,1)$.

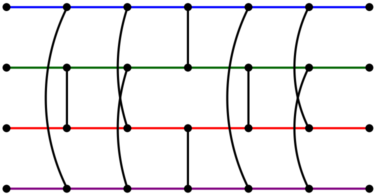
Splits of cycle structure





Splits of cycle structure





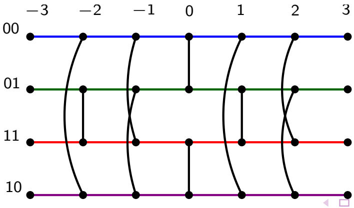
Splits of cycle structure

For a positive integer n divisible by 3, let $K = \mathbb{Z}_2^2$, let

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix},$$

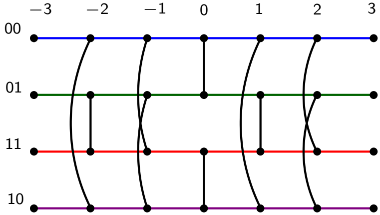
and let $e = (1, 0) \in K$. We define the graph $\mathrm{A}(n)$ by letting

$$\begin{aligned} \mathrm{V}(\mathrm{A}(n)) &= \mathbb{Z}_n \times K; \\ \mathrm{E}(\mathrm{A}(n)) &= \{\{(i,x),(i+1,x)\} : x \in K, i \in \mathbb{Z}_n\} \\ &\quad \cup \{(i,x),\{(i,x+eA^i)\} : x \in K, i \in \mathbb{Z}_n\}. \end{aligned}$$



CVT with girth 7

- Observe that $\mathrm{A}(n)$ is in fact equal to the Cayley graph $\mathrm{Cay}(G; \{(0, e), (1, 0)\}$ where $G = \mathbb{Z}_n \ltimes_\Theta \mathbb{Z}_2^2$ with $\Theta \colon \mathbb{Z}_n \to \mathrm{GL}(2, 2)$, $\Theta(i) = A^i$.
- Girth 7.
- Edge-signature $(4, 4, 6)$.
- No AT.
- $CVT[36, 2] = \mathrm{A}(9)$.



Splits of cycle structure

Let $\Gamma = \mathrm{R}_{2m}(m+2, m+1)$ for $m \geq 3$.

Structure	CL	Cond	Bipartite	$\lvert\mathrm{Aut}(\mathcal{Y})\rvert$	Γ'	$g(\Gamma')$	ESig	E
$\mathcal{Y}^*$	4	-	if m even	$2m(2^m)$	$\mathrm{SpPX}(m,2)$	4	$(0,1,1)$	
$\mathcal{Y}_{100*}$	m	$3\vert m$	Yes	$16m$		8	$(4,4,4)$	if m
$\mathcal{Y}'_{100*}$	$2m$	$3\vert m$	No	$16m$		8	$(4,4,4)$	if m
$\mathcal{Y}_{1100*}$	m	$4\vert m$	Yes	$16m$	trun. of maps of type $\{4,m\}$	8	$(1,1,2)$	Y
$\mathcal{Y}'_{1100*}$	$2m$	$4\vert m$	Yes	$16m$	trun. of maps of type $\{4,2m\}$	8	$(1,1,2)$	Y

Table: Cycle Structures in $R_{2m}(m+2, m+1)$

Splits of cycle structure

Let $\Gamma = \mathrm{R}_{2m}(m+2, m+1)$ for $m \geq 3$.

Structure	CL	Cond	Bipartite	$\lvert \mathrm{Aut}(\mathcal{Y}) \rvert$	Γ'	$g(\Gamma')$	ESig	Bip	Ty
$\mathcal{Y}^*$	4	-	if m even	$2m(2^m)$	$\mathrm{SpPX}(m, 2)$	4	$(0, 1, 1)$		
$\mathcal{Y}_{100*}$	m	$3 \vert m$	Yes	$16m$	Cover of $\mathrm{A}(4m)$	8	$(4, 4, 4)$	if m even	9
$\mathcal{Y}'_{100*}$	$2m$	$3 \vert m$	No	$16m$	Cover of $\mathrm{A}(4m)$	8	$(4, 4, 4)$	if m even	
$\mathcal{Y}_{1100*}$	m	$4 \vert m$	Yes	$16m$	Cover of $DP(m, 1)$	8	$(1, 1, 2)$	Yes	
$\mathcal{Y}'_{1100*}$	$2m$	$4 \vert m$	Yes	$16m$	Cover of $DP(m, 1)$	8	$(1, 1, 2)$	Yes	

Table: Cycle Structures in $R_{2m}(m+2, m+1)$

Splits of cycle structure

There are 27 non-isomorphic cycle structures relate to edge-transitive Rose Windows graphs, + BD

Most of the automorphism groups of such tetravalent graphs are 1-regular.

Cycle structure

Theorem

Let Γ be a tetravalent graph, $uv \in E(\Gamma)$ and $G \leq \mathrm{Aut}(\Gamma)$ acting regularly on the arcs of Γ. Let $\nu \in G$, such that $u^\nu = v$ and $v^\nu = u$, and let α, β, γ be the three non-identity elements of $H = G_u$. We consider two cases for the structure of H:

1. *H is cyclic, i.e., isomorphic to C_4. Then there is one orbit of G-chiral consistent cycles, one of G-reflexible consistent cycles,* **one cycle structure**, *and at most one semitransitive orientation. Further, there is one rotary map $\mathcal{M}$ with $G \leq \mathrm{Aut}^+(\mathcal{M})$. This map is G-chiral and its set of faces is the unique orbit of G-chiral consistent cycles. Moreover, Γ admits a semitransitive orientation relative to G if and only if the map $\mathcal{M}$ is face-bipartite .*
2. *H is not cyclic, so H is isomorphic to $C_2 \times C_2$. Then, relative to G, there are three orbits of reflexible consistent cycles (with the shunts $\alpha\nu$, $\beta\nu$ and $\gamma\nu$, respectively),* **three cycle structures**, *at most three semitransitive orientations and no rotary map with $\mathrm{Aut}(\mathcal{M}) \leq G$. If Δ is a*

Work in progress..

- Database for CVT.
- For all tetravalent edge-transitive bicirculant graphs:
  - Consistent cycles,
  - Cycle structure,
  - Splits,
  - Rotary (edge-transitive) Maps,
  - ...
- Classification of all finite connected cubic vertex-transitive tetracirculants (AT,GRR, $Type2^*$).
  The type 2^* once are splittings of tetravalent edge-transitive bicirculant graphs.
  Not all the CVT that are Sp of tetravalent edge-transitive bicirculant graphs are tetracirculants*
- Generalization of the splits..

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thank you! :)